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THE GRAND UNIFIED THEORY OF CLASSICAL QUANTUM MECHANICS

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1. INTRODUCTION

A theory of classical quantum mechanics (CQM), derived from first principles,[1] successfully applies physical laws on all scales. The classical wave equation is solved with the constraint that a bound electron cannot radiate energy. The mathematical formulation for zero radiation based on Maxwell's equations follows from a derivation by Haus.[2] The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. CQM gives closed form solutions for the atom, including the stability of the n=1 state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, **r x p**, can be applied directly to the wave function, called an orbitsphere (a current density function), that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations.

For or any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. By applying this condition to electromagnetic and gravitational fields at

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particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos. The Universe is time harmonically oscillatory in matter energy and spacetime expansion and contraction with a minimum radius that is the gravitational radius. In closed form equations with fundamental constants only, CQM gives the deflection of light by stars, the precession of the perihelion of Mercury, the particle masses, the Hubble constant, the age of the Universe, the observed acceleration of the expansion, the power of the Universe, the power spectrum of the Universe, the microwave background temperature, the uniformity of the microwave background radiation, the microkelvin spatial variation of the microwave background radiation, the observed violation of the GZK cutoff, the mass density, the large scale structure of the Universe, and the identity of dark matter which matches the criteria for the structure of galaxies. In a special case wherein the gravitational potential energy density of a blackhole equals that of the Plank mass, matter converts to energy and spacetime expands with the release of a gamma-ray burst. The singularity in the SM is eliminated.

2. COSMOLOGICAL THEORY BASED ON MAXWELL'S EQUATIONS

Maxwell's equations and special relativity are based on the law of propagation of a electromagnetic wave front in the form

$$1/c^2(\delta\omega/\delta t)^2 - \left[(\delta\omega/\delta x)^2 + (\delta\omega/\delta y)^2 + (\delta\omega/\delta\omega)^2\right] = 0 \quad (1)$$

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. Thus, the equation $1/c^2(\delta\omega/\delta t)^2 - (grad\omega)^2 = 0$ acquires a general character; it is more general than Maxwell's equations from which Maxwell originally derived it.

A discovery of the present work is that the classical wave equation governs: (1) the motion of bound electrons, (2) the propagation of any form of energy, (3) measurements between inertial frames of reference such as time, mass, momentum, and length (Minkowski tensor), (4) fundamental particle production and the conversion of matter to energy, (5) a relativistic correction of spacetime due to particle production or annihilation (Schwarzschild metric), (6) the expansion and contraction of the Universe, (7) the basis of the relationship between Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity.

The relationship between the time interval between ticks t of a clock in motion with velocity v relative to an observer and the time interval t_0 between ticks on a clock at rest relative to an observer [24] is

$$(ct)^2 = (ct_0)^2 + (vt)^2. \quad (2)$$

Thus, the time dilation relationship based on the constant maximum speed of light c in any inertial frame is $t = t_0\Big/\sqrt{1-\left(v^2/c^2\right)}$. The metric $g_{\mu\nu}$ for Euclidean space is the

Minkowski tensor $\eta_{\mu\nu}$. In this case, the separation of proper time between two events x^μ and $x^\mu + dx^\mu$ is $d\tau^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$.

3. THE EQUIVALENCE OF THE GRAVITATIONAL MASS AND THE INERTIAL MASS

The equivalence of the gravitational mass and the inertial mass m_g/m_i = universal constant which is predicted by Newton's law of mechanics and gravitation is experimentally confirmed to less 1 X 10^{-11}.[3] In physics, the discovery of a universal constant often leads to the development of an entirely new theory. From the universal constancy of the velocity of light c, the special theory of relativity was derived; and from Planck's constant h, the quantum theory was deduced. Therefore, the universal constant m_g/m_i should be the key to the gravitational problem. The energy equation of Newtonian gravitation is

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \text{constant} \tag{3}$$

Since h, the angular momentum per unit mass, is $h = L/m = |\mathbf{r} \times \mathbf{v}| = r_0 v_0 \sin\phi$, the eccentricity e may be written as

$$e = [1 + \left(v_0^2 - \frac{2GM}{r_0}\right)\frac{r_0^2 v_0^2 \sin^2\phi}{G^2M^2}]^{1/2}, \tag{4}$$

where m is the inertial mass of a particle, v_0 is the speed of the particle, r_0 is the distance of the particle from a massive object, ϕ is the angle between the direction of motion of the particle and the radius vector from the object, and M is the total mass of the object (including a particle). The eccentricity e given by Newton's differential equations of motion in the case of the central field permits the classification of the orbits according to the total energy E [4] (column 1) and the orbital velocity squared, v_0^2, relative to the gravitational velocity squared, $2GM/r_0$ [4] (column 2):

$E < 0$	$v_0^2 < 2GM/r_0$	$e < 1$	ellipse
$E < 0$	$v_0^2 < 2GM/r_0$	$e = 0$	circle (special case of ellipse)
$E = 0$	$v_0^2 = 2GM/r_0$	$e = 1$	parabolic orbit
$E > 0$	$v_0^2 > 2GM/r_0$	$e > 1$	hyperbolic orbit

4. CONTINUITY CONDITIONS FOR THE PRODUCTION OF A PARTICLE FROM A PHOTON TRAVELING AT LIGHT SPEED

A photon traveling at the speed of light gives rise to a particle with an initial radius equal to its Compton wavelength bar.

$$r = \lambda\!\!\!{}^-_C = \frac{\hbar}{m_0 c} = r_\alpha^*, \tag{5}$$

The particle must have an orbital velocity equal to Newtonian gravitational escape velocity v_g of the antiparticle.

$$v_g = \sqrt{\frac{2Gm}{r}} = \sqrt{\frac{2Gm_0}{\lambda\!\!\!{}^-_C}}. \tag{6}$$

The eccentricity is one. The orbital energy is zero. The particle production trajectory is a parabola relative to the center of mass of the antiparticle.

4.1 A Gravitational Field as a Front Equivalent to Light Wave Front

The particle with a finite gravitational mass gives rise to a gravitational field that travels out as a front equivalent to a light wave front. The form of the outgoing gravitational field front traveling at the speed of light is $f(t - r/c)$ and $d\tau^2$ is given by

$$d\tau^2 = f(r)dt^2 - \frac{1}{c^2}\left[f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{7}$$

The speed of light as a constant maximum as well as phase matching and continuity conditions of the electromagnetic and gravitational waves require the following form of the squared displacements:

$$(c\tau)^2 + (v_g t)^2 = (ct)^2, \tag{8}$$

$$f(r) = \left(1 - \left(\frac{v_g}{c}\right)^2\right). \tag{9}$$

In order that the wave front velocity does not exceed c in any frame, spacetime must undergo time dilation and length contraction due to the particle production event. *The derivation and result of spacetime time dilation is analogous to the derivation and result of special relativistic time dilation* wherein the relative velocity of two inertial frames replaces the gravitational velocity.

The general form of the metric due to the relativistic effect on spacetime due to mass m_0 with v_g given by Eq. (6) is

$$d\tau^2 = \left(1 - \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{10}$$

The gravitational radius, r_g, of each orbitsphere of the particle production event, each of

mass m_0 and the corresponding general form of the metric are respectively

$$r_g = \frac{2Gm_0}{c^2}, \tag{11}$$

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{12}$$

Masses and their effects on spacetime *superimpose*. The separation of proper time between two events x^μ and $x^\mu + dx^\mu$ is

$$d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \tag{13}$$

The metric $g_{\mu\nu}$ for non-Euclidean space due to the relativistic effect on spacetime due to mass M is *the Schwarzschild metric which gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity.*

4.2. Particle Production Continuity Conditions from Maxwell's Equations, and the Schwarzchild Metric

The photon to particle event requires a transition state that is continuous wherein the velocity of a transition state orbitsphere is the speed of light. The radius, r, is the Compton wavelength bar, λ_c, given by Eq. (5). At production, the Planck equation energy, the electric potential energy, and the magnetic energy are equal to m_0c^2.

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the masses of fundamental particles. Substitution of $r = \bar{\lambda}_C$; $dr = 0$; $d\theta = 0$; $\sin^2\theta = 1$ into the Schwarzschild metric gives

$$d\tau = dt\left(1 - \frac{2Gm_0}{c^2 r_\alpha^*} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}, \tag{14}$$

with $v^2 = c^2$, the relationship between the proper time and the coordinate time is

$$\tau = ti\sqrt{\frac{2GM}{c^2 r_\alpha^*}} = ti\sqrt{\frac{2GM}{c^2 \bar{\lambda}_c}} = ti\frac{v_g}{c}. \tag{15}$$

When the orbitsphere velocity is the speed of light, continuity conditions based on the constant maximum speed of light given by Maxwell's equations are mass energy = Planck equation energy = electric potential energy = magnetic energy = mass/spacetime metric energy. Therefore, $m_o c^2 = \hbar\omega^* = V = E_{mag} = E_{spacetime}$

$$m_0c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0\lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_0)^2\lambda_C^3} = \frac{\alpha h}{1\sec}\sqrt{\frac{\lambda_C c^2}{2Gm}} \; . \tag{16}$$

The continuity conditions based on the constant maximum speed of light given by the Schwarzschild metric are:

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}} \; , \tag{17}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2\lambda_C}}}{\alpha} = i\frac{v_g}{\alpha c} \; . \tag{18}$$

5. MASSES OF FUNDAMENTAL PARTICLES

Each of the Planck equation energy, electric energy, and magnetic energy corresponds to a particle given by the relationship between the proper time and the coordinate time. The electron and down-down-up neutron correspond to the Planck equation energy. The muon and strange-strange-charmed neutron correspond to the electric energy. The tau and bottom-bottom-top neutron correspond to the magnetic energy. The particle must possess the escape velocity v_g relative to the antiparticle where $v_g<c$. According to Newton's law of gravitation, the eccentricity is one and the particle production trajectory is a parabola relative to the center of mass of the antiparticle.

5.1. The Electron-Antielectron Lepton Pair

A clock is defined in terms of a self consistent system of units used to measure the particle mass. The proper time of the particle is equated with the coordinate time according to the Schwarzschild metric corresponding to light speed. The special relativistic condition corresponding to the Planck energy gives the mass of the electron.

$$2\pi\frac{\hbar}{mc^2} = sec\sqrt{\frac{2Gm^2}{c\alpha^2\hbar}} \; , \tag{19}$$

$$m_e = \left(\frac{h\alpha}{sec\, c^2}\right)^{\frac{1}{2}}\left(\frac{c\hbar}{2G}\right)^{\frac{1}{4}} = 9.1097\, X\, 10^{-31}\, kg \; , \tag{20}$$

$$m_e = 9.1097\, X\, 10^{-31} kg - 18\, eV/c^2\, (\nu_e) = 9.1094\, X\, 10^{-31} kg \; , \tag{21}$$

$$m_{e\,\text{experimental}} = 9.1095\, X\, 10^{-31} kg \; . \tag{22}$$

5.2. Down-Down-Up Neutron (DDU)

The corresponding equation for production of the neutron is

7. RELATIONSHIP OF MATTER TO ENERGY AND SPACETIME EXPANSION

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime. The limiting velocity c results in the contraction of spacetime due to particle production, which is given by $2\pi r_g$ where r_g is the gravitational radius of the particle. This has implications for the expansion of spacetime when matter converts to energy. Q the mass/energy to expansion/contraction quotient of spacetime is given by the ratio of the mass of a particle at production divided by T the period of production.

$$Q = \frac{m_0}{T} = \frac{m_0}{\frac{2\pi r_g}{c}} = \frac{m_0}{\frac{2\pi \frac{2Gm_0}{c^2}}{c}} = \frac{c^3}{4\pi G} = 3.22 X 10^{34} \frac{kg}{sec}. \tag{34}$$

The gravitational equations with the equivalence of the particle production energies [Eq. (16)] permit the conservation of mass/energy ($E=mc^2$) and spacetime ($c^3/4\pi G=3.22$ X 10^{34} kg/sec). With the conversion of 3.22 X 10^{34} kg of matter to energy, spacetime expands by 1 sec. The photon has inertial mass and angular momentum, but due to Maxwell's equations and the implicit special relativity it does not have a gravitational mass.

7.1. Cosmological Consequences

The Universe is closed (it is *finite but with no boundary*). It is a 3-sphere Universe-Riemannian three dimensional hyperspace plus time of constant positive curvature at each r-sphere. The *Universe is oscillatory in matter/energy and spacetime* with a finite minimum radius, the gravitational radius. Spacetime expands as mass is released as energy which provides the basis of the atomic, thermodynamic, and cosmological arrows of time. Different regions of space are isothermal even though they are separated by greater distances than that over which light could travel during the time of the expansion of the Universe.[9] Presently, stars and large scale structures exist which are older than the elapsed time of the present expansion as stellar, galaxy, and supercluster evolution occurred during the contraction phase.[10–16] The maximum power radiated by the Universe which occurs at the beginning of the expansion phase is $P_U=c^5/4\pi G = 2.89\ X\ 10^{51} W$. Observations beyond the beginning of the expansion phase are not possible since the Universe is entirely matter filled.

7.2. The Period of Oscillation of the Universe Based on Closed Propagation of Light

Mass/energy is conserved during harmonic expansion and contraction. The gravitational potential energy E_{grav} given by Eq. (28) with $m_0 = m_U$ is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G. The gravitational velocity v_G [Eq. (30) with $r=r_G$ and $m_0 = m_U$] is the speed of light in a circular orbit wherein the eccentricity is equal to zero and the escape velocity from the Universe can never be reached. The period of the oscillation of the Universe and the period for light to transverse the Universe corresponding to the gravitational radius r_G must be equal. The harmonic oscillation period, T, is

$$T = \frac{2\pi r_G}{c} = \frac{2\pi G m_U}{c^3} = \frac{2\pi G(2\,X\,10^{54}\,kg)}{c^3} = 3.10\,X\,10^{19}\,sec = 9.83\,X\,10^{11}\,\text{years}, \tag{35}$$

where the mass of the Universe, m_U, is approximately 2 X $10^{54}kg$. (The initial mass of the Universe of 2 X $10^{54}kg$ is based on internal consistency with the size, age, Hubble constant, temperature, density of matter, and power spectrum.) Thus, the observed Universe will expand as mass is released as photons for 4.92 X 10^{11}*years*. At this point in its world line, the Universe will obtain its maximum size and begin to contract.

8. THE DIFFERENTIAL EQUATION OF THE RADIUS OF THE UNIVERSE

Based on conservation of mass/energy ($E=mc^2$) and spacetime ($c^3/4\pi G$=3.22 X 10^{34} *kg/sec*). The Universe behaves as a simple harmonic oscillator having a restoring force, **F**, which is proportional to the radius. The proportionality constant, k, is given in terms of the potential energy, E, gained as the radius decreases from the maximum expansion to the minimum contraction.

$$\frac{E}{\aleph^2} = k. \tag{36}$$

Since the gravitational potential energy E_{grav} is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G

$$F = -k\aleph = -\frac{m_U c^2}{r_G^2}\aleph = -\frac{m_U c^2}{\left(\frac{Gm_U}{c^2}\right)^2}\aleph. \tag{37}$$

And, the differential equation of the radius of the Universe, $\aleph$ is

$$m_U\ddot{\aleph} + \frac{m_U c^2}{r_G^2}\aleph = m_U\ddot{\aleph} + \frac{m_U c^2}{\left(\frac{Gm_U}{c^2}\right)^2}\aleph = 0. \tag{38}$$

The *maximum radius of the Universe*, the amplitude, r_0, of the time harmonic variation in the radius of the Universe, is given by the quotient of the total mass of the Universe and Q, the mass/energy to expansion/contraction quotient.

$$r_0 = \frac{m_U}{Q} = \frac{m_U}{\frac{c^3}{4\pi G}} = \frac{2\,X\,10^{54}\,kg}{\frac{c^3}{4\pi G}} = 1.97\,X\,10^{12}\,\textit{light years}. \tag{39}$$

The *minimum radius* which corresponds to the gravitational radius r_g, given by Eq. (11) with $m_0=m_U$ is 3.12 X 10^{11} *light years.* When the radius of the Universe is the gravitational radius, r_g, the proper time is equal to the coordinate time by Eq. (15) and the gravitational escape velocity v_g of the Universe is the speed of light. The radius of the Universe as a function of time is

$$\aleph = \left(r_g + \frac{cm_U}{Q}\right) - \frac{cm_U}{Q}\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right) = \left(\frac{2Gm_U}{c^2} + \frac{\frac{cm_U}{c^3}}{4\pi G}\right) - \frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right). \tag{40}$$

The expansion/contraction rate, $\dot{\aleph}$, is given by time derivative of Eq. (40)

$$\dot{\aleph} = 4\pi c X 10^{-3} \sin\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\frac{km}{sec}. \tag{41}$$

9. THE HUBBLE CONSTANT

The *Hubble constant* is given by the ratio of the expansion rate given in units of *km/sec* divided by the radius of the expansion in *Mpc*. The radius of expansion is equivalent to the radius of the light sphere with an origin at the time point when the Universe stopped contracting and started to expand.

$$H = \frac{\dot{\aleph}}{t\,Mpc} = \frac{4\pi c X 10^{-3} \sin\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\frac{km}{sec}}{t\,Mpc}, \tag{42}$$

for $t = 10^{10}$ *light years* $= 3.069\,X\,10^{3}\,Mpc$, the Hubble constant, H_0, is 78.6 *km/sec·Mpc*. The experimental value is[17] $H_0 = 80 \pm 17$ *km/sec·Mpc*.

10. THE DENSITY OF THE UNIVERSE AS A FUNCTION OF TIME

The density of the Universe as a function of time $\rho_U(t)$ is given by the ratio of the mass as a function of time and the volume as a function of time.

$$\rho_U(t) = \frac{m_U(t)}{V(t)} = \frac{m_U(t)}{\frac{4}{3}\pi\aleph(t)^3} = \frac{\frac{m_U}{2}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)}{\frac{4}{3}\pi\left(\left(\frac{2Gm_U}{c^2} + \frac{\frac{cm_U}{c^3}}{4\pi G}\right) - \frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)^3}, \tag{43}$$

for $t = 10^{10}$ *light years*, $\rho_U = 1.7\,X\,10^{-32}\,g/cm^3$ The density of luminous matter of stars and gas of galaxies is about $\rho_U = 2\,X\,10^{-31}\,g/cm^3$.[18-19]

11. THE POWER OF THE UNIVERSE AS A FUNCTION OF TIME, $P_U(t)$

From $E = mc^2$ and Eq. (34),

$$P_U(t)=\frac{c^5}{8\pi G}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)\right) \tag{44}$$

For $t = 10^{10}$ *light years*, $P_U(t)= 2.88\,X\,10^{51}\,W$. The observed power is consistent with that predicted.

12. THE TEMPERATURE OF THE UNIVERSE AS A FUNCTION OF TIME

The temperature of the Universe as a function of time, $T_U(t)$, follows from the Stefan-Boltzmann law.

$$T_U(t)=\left(\frac{1}{1+\frac{Gm_U(t)}{c^2\aleph(t)}}\right)\left[\frac{R_U(t)}{e\sigma}\right]^{\frac{1}{4}}=\left(\frac{1}{1+\frac{Gm_U(t)}{c^2\aleph(t)}}\right)\left[\frac{\frac{P_U(t)}{4\pi\aleph(t)^2}}{e\sigma}\right]^{\frac{1}{4}}. \tag{45}$$

The calculated uniform temperature is about 2.7 K which is in agreement with the observed microwave background temperature.[9]

13. POWER SPECTRUM OF THE COSMOS

The power spectrum of the cosmos, as measured by the Las Campanas Survey, generally follows the prediction of cold dark matter on the scales of 200 million to 600 million light-years. However, the power increases dramatically on scales of 600 million to 900 million light-years. The infinitesimal temporal displacement, $d\tau^2$, is given by Eq. (13).

The relationship between the proper time and the coordinate time is

$$\tau = t\sqrt{1-\frac{2Gm_U}{c^2 r}} = t\sqrt{1-\frac{r_g}{r}}. \tag{46}$$

The power maximum in the proper frame occurs at

$$\tau = 5\,X\,10^9 \textit{ light years}\sqrt{1-\frac{3.12\,X\,10^{11}\textit{ light years}}{3.22\,X\,10^{11}\textit{ light years}}} = 880\,X\,10^6 \textit{ light years}. \tag{47}$$

The power maximum of the current observable Universe is predicted to occur on the

scale of $880\ X\ 10^6$ *light years*. There is excellent agreement between the predicted value and the experimental value of $600\text{–}900\ X\ 10^6$ *light years*.[16]

14. THE EXPANSION/CONTRACTION ACCELERATION, $\ddot{\aleph}$

The expansion/contraction acceleration rate, $\ddot{\aleph}$, is given by the time derivative of Eq. (41).

$$\ddot{\aleph} = 2\pi \frac{c^4}{Gm_U} \cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3} sec}\right) = \ddot{\aleph} = H_o = 78.7 \cos\left(\frac{2\pi t}{3.01\ X\ 10^5\ Mpc}\right) \frac{km}{sec \cdot Mpc} \tag{48}$$

The differential in the radius of the Universe, $\Delta\aleph$, due to its acceleration is given by $\Delta\aleph = 1/2\ddot{\aleph}t^2$. The differential in expanded radius for the elapsed time of expansion, $t = 10^{10}$ *light years* corresponds to a decease in brightness of a supernovae standard candle of about an order of magnitude of that expected where the distance is taken as $\Delta\aleph$. This result based on the predicted rate of acceleration of the expansion is consistent with the experimental observation.[20–22]

Furthermore, the microwave background radiation image obtained by the Boomerang telescope[23] is consistent with a Universe of nearly flat geometry since the commencement of its expansion. The data is consistent with a large offset radius of the Universe with a fractional increase in size since the commencement of expansion about 10 billion years ago.

15. THE PERIODS OF SPACETIME EXPANSION/CONTRACTION AND PARTICLE DECAY/PRODUCTION FOR THE UNIVERSE ARE EQUAL

The period of the expansion/contraction cycle of the radius of the Universe, T, is given by Eq. (35). It follows from the Poynting power theorem with spherical radiation that the transition lifetimes are given by the ratio of energy and the power of the transition.

$$\tau = \frac{energy}{power} = \frac{[\hbar\omega]}{\left[\frac{2\pi c}{[(2l+1)!!]}\left(\frac{l+1}{l}\right)k^{2l+1}\left|Q_{lm} + Q'_{lm}\right|^2\right]}$$

$$= \frac{1}{2\pi}\left(\frac{h}{e^2}\right)\sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{[(2l+1)!!]}{2\pi}\left(\frac{l}{l+1}\right)\left(\frac{l+3}{3}\right)^2 \frac{1}{(kr_n)^{2l}\omega} \tag{49}$$

Exponential decay applies to electromagnetic energy decay $h(t) = e^{-\frac{1}{T}t}u(t)$. The

coordinate time is imaginary because energy transitions are spacelike due spacetime expansion from matter to energy conversion. For example, the mass of the electron (a fundamental particle) is given by

$$\frac{2\pi\lambda_C}{\sqrt{\frac{2Gm_e}{\lambda_C}}} = \frac{2\pi\lambda_C}{v_g} = i\alpha^{-1}\,sec, \tag{50}$$

where v_g is Newtonian gravitational velocity [Eq. (6)]. When the gravitational radius r_g is the radius of the Universe, the proper time is equal to the coordinate time by Eq. (15), and the gravitational escape velocity v_g of the Universe is the speed of light. Replacement of the coordinate time, t, by the spacelike time, it, gives

$$h(t) = Re\left[e^{-i\frac{1}{T}t}\right] = cos\frac{2\pi}{T}t, \tag{51}$$

where the period is T [Eq. (35)]. The continuity conditions based on the constant maximum speed of light (Maxwell's equations) are given by Eqs. (16). The continuity conditions based on the constant maximum speed of light (Schwarzschild metric) are given by Eqs. (17–18). The periods of spacetime expansion/contraction and particle decay/production for the Universe are equal because only the particles which satisfy Maxwell's equations and the relationship between proper time and coordinate time imposed by the Schwarzschild metric may exist.

16. WAVE EQUATION

The general form of the light front wave equations is given by Eq. (1). The equation of the radius of the Universe, $\aleph$, may be written as

$$\aleph = \left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}} cos\left(\frac{2\pi}{\frac{2\pi Gm_U}{c^3}sec}\left(t - \frac{\aleph}{c}\right)\right) m, \tag{52}$$

which is a solution of the wave equation for a light wave front.

17. CONCLUSION

Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity are unified.

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The Grand Unified Theory of Classical Quantum Mechanics

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Presented at
the Global Foundation, Inc.
The Role of Attractive and Repulsive Gravitational Forces
in Cosmic Acceleration of Particles
The Origin of the Cosmic Gamma Ray Bursts
(29th Conference
on High Energy Physics and Cosmology Since 1964)
Ft. Lauderdale, FL
Lago Mar Resort
December 14-17, 2000
Dr. Behram Kursunoglu, Chairman

The Grand Unified Theory of Classical Quantum Mechanics

A theory of classical quantum mechanics (CQM) is derived from first principles that successfully applies physical laws on all scales. Using the classical wave equation with the constraint of nonradiation based on Maxwell's equations, CQM gives closed form solutions for the atom including the stability of the n=1 state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations.

For or any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave. By applying this condition to electromagnetic and gravitational fields at particle production, the Schwarzschild metric (SM) is derived from the classical wave equation which modifies general relativity to include conservation of spacetime in addition to momentum and matter/energy. The result gives a natural relationship between Maxwell's equations, special relativity, and general relativity. It gives gravitation from the atom to the cosmos. The universe is time harmonically oscillatory in matter energy and spacetime expansion and contraction with a minimum radius that is the gravitational radius. In closed form equations with fundamental constants only, CQM gives the deflection of light by stars, the precession of the perihelion of Mercury, the particle masses, the Hubble constant, the age of the universe, the observed acceleration of the expansion, the power of the universe, the power spectrum of the universe, the microwave background temperature, the uniformity of the microwave background radiation, the microkelvin spatial variation of the microwave background radiation, the observed violation of the GZK cutoff, the mass density, the large scale structure of the universe, and the identity of dark matter which matches the criteria for the structure of galaxies. In a special case wherein the gravitational potential energy density of a blackhole equals that of the Plank mass, matter converts to energy and spacetime expands with the release of a gamma ray burst. The singularity in the SM is eliminated.

The Grand Unified Theory of Classical Quantum Mechanics

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Wave Equation

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right]\rho(r,\theta,\phi,t) = 0$$

Boundary Constraint from Maxwell's Equations

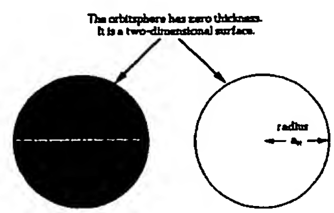
For non-radiative states, the current-density function must NOT possess spacetime Fourier components that are synchronous with waves traveling at the speed of light.

The solution for the radial function which satisfies the boundary condition is a delta function

$$f(r) = \frac{1}{r^2}\delta(r - r_n)$$

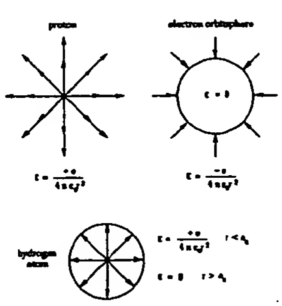
where $r_n = nr_1$

The Orbitsphere



The **orbitsphere** is a two dimensional spherical shell with the Bohr radius of the hydrogen atom.

Electric Fields of Proton, Electron, and Hydrogen Atom



The Wavelength of the Free Electron

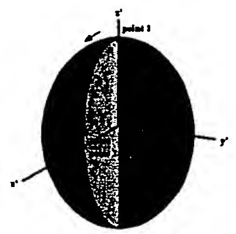
$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n}$$

$$v_n = \frac{\hbar}{m_e r_n}$$

$$\sum |\mathbf{L}_i| = \sum |\mathbf{r} \times m_i \mathbf{v}| = m_e r_n \frac{\hbar}{m_e r_n} = \hbar$$

the de Broglie wavelength is observed for the electron

Spin Function



Two infinitesimal point masses (charges) of two orthogonal great circle current loops in the orbitsphere frame.

Time Dependence of Representative Point of Each Great Circle Current Loop

point one:

$$x_1' = 0 \qquad y_1' = -r_n \sin(\omega_n t) \qquad z_1' = r_n \cos(\omega_n t)$$

point two:

$$x_2' = r_n \cos(\omega_n t) \qquad y_2' = 0 \qquad z_2' = r_n \sin(\omega_n t)$$

Nested Series of Great Circle Current Loops

point one:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha) \\ 0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha) \\ \sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_1' \\ y_1' \\ z_1' \end{bmatrix}$$

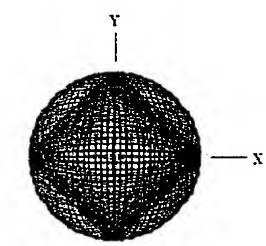
and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$

point two:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha) \\ 0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha) \\ \sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha) \end{bmatrix} \begin{bmatrix} x_2' \\ y_2' \\ z_2' \end{bmatrix}$$

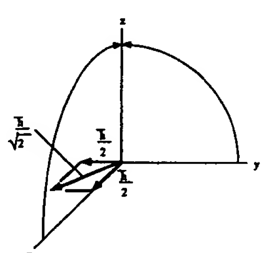
and $\Delta\alpha' = -\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$

Current Pattern of the Orbitsphere

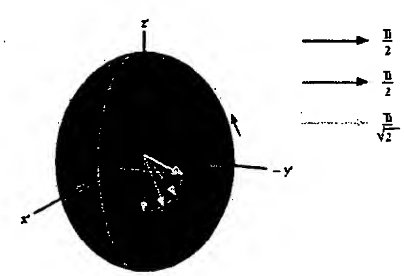


VIEW ALONG THE Z AXIS

Resultant Angular Momentum Vector



Projection of the Resultant Angular Momentum Vector over the Orbitsphere



Stern-Gerlach Experiment

Spin Quantum Number, s $\left(s=\frac{1}{2}; m_s=\pm\frac{1}{2}\right)$

$$\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}} \Delta\alpha = \sqrt{2}\pi$$

$$\langle \mathbf{L}_{xy}\rangle_{\Sigma\Delta\alpha} = \frac{1}{2}\frac{\hbar}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\hbar}{4}$$

$$\langle \mathbf{L}_z\rangle_{\Sigma\Delta\alpha} = \frac{1}{2}\frac{\hbar}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\hbar}{4}$$

$$\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}} |\Delta\alpha'| = \sqrt{2}\pi$$

$$\langle \mathbf{L}_{xy}\rangle_{\Sigma\Delta\alpha'} = 0$$

$$\langle \mathbf{L}_z\rangle_{\Sigma\Delta\alpha'} = \frac{1}{2}\frac{\hbar}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\hbar}{4}$$

Total Component Angular Momentum

$$\langle \mathbf{L}_{xy}\rangle_{\Sigma\Delta\alpha} = \frac{\hbar}{4} + 0 = \frac{\hbar}{4}$$

$$\langle \mathbf{L}_z\rangle_{\Sigma\Delta\alpha} = \frac{\hbar}{4} + \frac{\hbar}{4} = \frac{\hbar}{2}$$

Projection of the Angular Momentum onto a Vector S that Precesses about the Z-Axis

S the projection of the orbitsphere angular momentum that precesses about the z-axis called the spin axis at an angle of $\theta = \frac{\pi}{3}$ and an angle of $\phi = \pi$ with respect to $\langle \mathbf{L}_{xy}\rangle_{\Sigma\Delta\alpha}$ is

$$\mathbf{S} = \pm\sqrt{\frac{3}{4}}\hbar$$

S rotates about the z-axis at the Larmor frequency

$\langle \mathbf{S}_z\rangle$, the time averaged projection of the orbitsphere angular momentum onto the axis of the applied magnetic field is

$$\langle \mathbf{L}_z\rangle_{\Sigma\Delta\alpha} \pm \frac{\hbar}{2}$$

Electron g Factor

Conservation of angular momentum of the orbitsphere permits a discrete change of its "kinetic angular momentum" $(\mathbf{r}\times m\mathbf{v})$ by the the applied magnetic field of $\frac{\hbar}{2}$, and concomitantly the "potential angular momentum" $(\mathbf{r}\times e\mathbf{A})$ must change by $-\frac{\hbar}{2}$.

$$\Delta\mathbf{L} = \frac{\hbar}{2} - \mathbf{r}\times e\mathbf{A}$$

$$= \frac{\hbar}{2} - \frac{e\phi}{2\pi}$$

In order that the change of angular momentum, $\Delta\mathbf{L}$, equals zero, ϕ must be $\Phi_0 = \frac{h}{2e}$, the magnetic flux quantum.

Electron g Factor cont'd

The magnetic moment of the electron is parallel or antiparallel to the applied field only.

The total energy of the flip transition is the sum of the energy of a fluxon treading the orbitsphere and the energy of reorientation of the magnetic moment.

$$\Delta E_{mag}^{spin} = 2\left(\mu_B\mathbf{B} + \frac{\alpha}{2\pi}\mu_B\mathbf{B}\right)$$

$$\Delta E_{mag}^{spin} = 2\left(1 + \frac{\alpha}{2\pi}\right)\mu_B\mathbf{B}$$

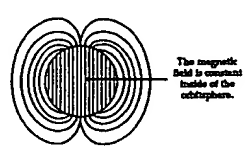
$$\Delta E_{mag}^{spin} = 2g\mu_B\mathbf{B}$$

The spin-flip transition can be considered as involving a magnetic moment of g times that of a Bohr magneton. The predicted value of the g factor is 1.00116. The experimental value is 1.00116.

Magnetic Fields of the Electron

$$\mathbf{H} = \frac{e\hbar}{m_e r_n^3}(\mathbf{i}_r\cos\theta - \mathbf{i}_\theta\sin\theta) \quad \text{for } r < r_n$$

$$\mathbf{H} = \frac{e\hbar}{2m_e r^3}(\mathbf{i}_r 2\cos\theta - \mathbf{i}_\theta\sin\theta) \quad \text{for } r > r_n$$



Derivation of the Magnetic Energy

The energy stored in the magnetic field of the electron is

$$E_{mag} = \frac{1}{2}\mu_o \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} H^2 r^2 \sin\theta dr d\theta d\Phi$$

$$E_{mag\ total} = \frac{\pi\mu_o e^2\hbar^2}{m_e^2 r_1^3}$$

Angular Functions

Based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta,\phi,t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right]A(\theta,\phi,t) = 0$$

where $\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r-r_n)A(\theta,\phi,t)$

and $A(\theta,\phi,t) = Y(\theta,\phi)k(t)$

$$\left[\frac{1}{r^2\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta}{\delta\theta}\right)_{r,\phi} + \frac{1}{r^2\sin^2\theta}\left(\frac{\delta^2}{\delta\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\delta^2}{\delta t^2}\right]A(\theta,\phi,t) = 0$$

where v is the linear velocity of the electron

Charge-Density Functions

The charge-density functions including the time-function factor are

$\ell = 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r-r_n)]\left[Y_\ell^m(\theta,\phi) + Y_0^0(\theta,\phi)\right]$$

$\ell \neq 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\}\right]$$

where

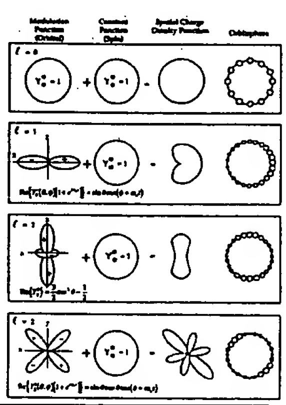
$$\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\} = \mathrm{Re}\left[Y_\ell^m(\theta,\phi) + Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right] = P_\ell^m(\cos\theta)\cos m\phi + P_\ell^m(\cos\theta)\cos(m\phi + \omega_n t)$$

and $\omega_n = 0$ for $m = 0$

Spin and Orbital Parameters

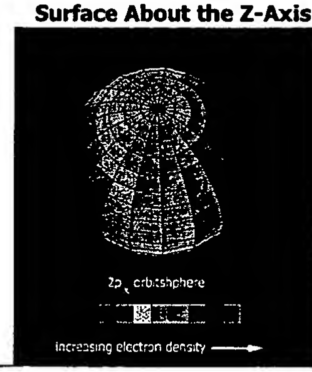
- The constant spin function is modulated by a time and spherical harmonic function.
- The modulation or traveling charge density wave corresponds to an orbital angular momentum in addition to a spin angular momentum.
- These states are typically referred to as p, d, f, etc. states or orbitals and correspond to an ℓ quantum number not equal to zero.

Orbital and Spin Functions

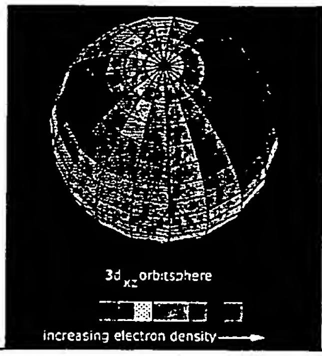


The orbital function modulates the constant (spin) function. (shown for t=0; cross-sectional view)

Charge Density Wave Moves on the Surface About the Z-Axis



Charge Density Wave Moves on the Surface About the Z-Axis



Moment of Inertia and Spin and Rotational Energies

$$\ell = 0$$

$$I_z = I_{spin} = \frac{m_e r_n^2}{2}$$

$$L_z = I\omega \mathbf{i}_z = \pm\frac{\hbar}{2}$$

$$E_{rotational} = E_{rotational,\ spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{2}\left[\frac{m_e r_n^2}{2}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{4}\left[\frac{\hbar^2}{2I_{spin}}\right]$$

Moment of Inertia and Spin and Rotational Energies cont'd

$$\ell \neq 0$$

$$I_{orbital} = m_e r_n^2\left[\frac{\ell(\ell+1)}{\ell^2+\ell+1}\right]^{\frac{1}{2}}$$

$$L_z = m\hbar$$

$$E_{rotational,\ orbital} = \frac{\hbar^2}{2I}\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right]$$

$$T = \frac{\hbar^2}{2m_e r_n^2}$$

$$\langle E_{rotational,\ orbital}\rangle = 0$$

Nonradiation Condition (Acceleration Without Radiation)

$$K(s,\Theta,\Phi,\omega) = 4\pi\omega_n \frac{\sin(2s_n r_n)}{2s_n r_n} \otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Theta)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Theta)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon}$$

$$\otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Phi)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Phi)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon} \frac{1}{4\pi}[\delta(\omega-\omega_n)+\delta(\omega+\omega_n)]$$

$$\mathbf{s}_n \bullet \mathbf{v}_n = \mathbf{s}_n \bullet \mathbf{c} = \omega_n$$

$$r_n = \lambda_n$$

Spacetime harmonics of $\frac{\omega_n}{c} = k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}} = k$ for which which the Fourier transform of the current-density function is nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met.

Force Balance Equation

$$\frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2}\frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2}\frac{\hbar^2}{m r_n^3}$$

$$r_1 = \frac{a_H}{Z}$$

Energy Calculations

- Potential Energy

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2e^2}{4\pi\varepsilon_o a_H} = -Z^2 X\ 4.3675\ X\ 10^{-18}\ J = -Z^2 X\ 27.2\ eV$$

- Kinetic Energy

$$T = \frac{Z^2e^2}{8\pi\varepsilon_o a_H} = Z^2 X\ 13.59\ eV \qquad T = E_{ele} = -\frac{1}{2}\varepsilon_o\int_{\infty}^{r_1}\mathbf{E}^2 dv$$

where $\mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2}$

- Electric Energy

$$E_{ele} = -\frac{Z^2e^2}{8\pi\varepsilon_o a_H} = -Z^2 X\ 2.1786\ X\ 10^{-18}\ J = -Z^2 X\ 13.598\ eV$$

Some Calculated Parameters for the Hydrogen Atom (n=1)

radius	$r_1 = a_H$	$5.2918 \times 10^{-11}\ m$
potential energy	$V = \frac{-e^2}{4\pi\varepsilon_o a_H}$	$-27.196\ eV$
kinetic energy	$T = \frac{e^2}{8\pi\varepsilon_o a_H}$	$13.598\ eV$
angular velocity (spin)	$\omega_1 = \frac{\hbar}{m_e r_1^2}$	$4.13 \times 10^{16}\ rads^{-1}$
linear velocity	$v_1 = r_1\omega_1$	$2.19 \times 10^{6}\ ms^{-1}$
wavelength	$\lambda_1 = 2\pi r_1$	$3.325 \times 10^{-10}\ m$
spin quantum number	$s = \frac{1}{2}$	$\frac{1}{2}$
moment of inertia	$I = m_e r_1^2 \sqrt{s(s+1)}$	$2.209 \times 10^{-41}\ kgm^2$
angular kinetic energy	$E_{angular} = \frac{1}{2} I \omega_1^2$	$11.78\ eV$

Some Calculated Parameters for the Hydrogen Atom (n=1) cont'd

magnitude of the angular momentum	$\hbar$	$1.0545 \times 10^{-34}\ Js$
projection of the angular momentum onto the S-axis	$S = \hbar\sqrt{s(s+1)}$	$9.133 \times 10^{-35}\ Js$
projection of the angular momentum onto the z-axis	$S_z = \frac{\hbar}{2}$	$5.273 \times 10^{-35}\ Js$
mass density	$\frac{m_e}{4\pi r_1^2}$	$2.589 \times 10^{-11}\ kgm^{-2}$
charge-density	$\frac{e}{4\pi r_1^2}$	$4.553\ Cm^{-2}$

Calculated Energies (Non-Relativistic) and Calculated Ionization Energies for Some One-electron Atoms

Atom	Calculated r_1 (a_o)	Calculated Kinetic Energy (eV)	Calculated Potential Energy (eV)	Calculated Ionization Energy (eV)	Experimental Ionization Energy (eV)
H	1.000	13.59	−27.18	13.59	13.59
He^{+}	0.500	54.35	−108.70	54.35	54.58
Li^{2+}	0.333	122.28	−244.56	122.28	122.45
Be^{3+}	0.250	217.40	−434.80	217.40	217.71
B^{4+}	0.200	339.68	−679.36	339.68	340.22
C^{5+}	0.167	489.14	−978.28	489.14	489.98
N^{6+}	0.143	665.77	−1331.54	665.77	667.03
O^{7+}	0.125	869.58	−1739.16	869.58	871.39

Excited States

- The orbitsphere is a dynamic spherical resonator cavity which traps photons of discrete frequencies.
- The relationship between an allowed radius and the "photon standing wave" wavelength is $2\pi r = n\lambda$ where n is an integer.
- The relationship between an allowed radius and the electron wavelength is $2\pi r = n\lambda$ where n=1,2,3,4,...
- The radius of an orbitsphere increases with the absorption of electromagnetic energy.
- The solutions to Maxwell's equations for modes that can be excited in the orbitsphere resonator cavity give rise to four quantum numbers, and the energies of the modes are the experimentally known hydrogen spectrum.

Excited States cont'd

The relationship between the electric field equation and the "trapped photon" source charge-density function is given by Maxwell's equation in two dimensions

$$\mathbf{n} \bullet (\mathbf{E}_1 - \mathbf{E}_2) = \frac{\sigma}{\varepsilon_o}$$

The photon standing electromagnetic wave is phase matched to with the electron

$$\mathbf{E}_{r\,photon\,n,l,m} = \frac{e(na_H)^l}{4\pi\varepsilon_o}\frac{1}{r^{(l+2)}}\left[-Y_0^0(\theta,\phi) + \frac{1}{n}\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_l^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\}\right]\right]$$

$\omega_n = 0$ for $m = 0$

$l = 1, 2, ..., n-1$

$m = -l, -l+1, ..., 0, ..., +l$

$$\mathbf{E}_{r\,total} = \frac{e}{4\pi\varepsilon_o r^2} + \frac{e(na_H)^l}{4\pi\varepsilon_o}\frac{1}{r^{(l+2)}}\left[-Y_0^0(\theta,\phi) + \frac{1}{n}\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_l^m(\theta,\phi)\left[1+e^{i\omega_n t}\right]\right\}\right]\right]$$

$\omega_n = 0$ for $m = 0$

For $r=na_H$ and $m=0$, the total radial electric field is

$$\mathbf{E}_{r\,total} = \frac{1}{n}\frac{e}{4\pi\varepsilon_o (na_H)^2}$$

Photon Absorption

- The energy of the photon which excites a mode in the electron spherical resonator cavity from radius a_H to radius na_H is

$$E_{photon} = \frac{e^2}{8\pi\varepsilon_o a_H}\left[1 - \frac{1}{n^2}\right] = h\nu = \hbar\omega$$

- The change in angular velocity of the orbitsphere for an excitation from n=1 to n=n is

$$\Delta\omega = \frac{\hbar}{m_e (a_H)^2} - \frac{\hbar}{m_e (na_H)^2} = \frac{\hbar}{m_e (a_H)^2}\left[1 - \frac{1}{n^2}\right]$$

- The kinetic energy change of the transition is

$$\frac{1}{2} m_e (\Delta v)^2 = \frac{e^2}{8\pi\varepsilon_o a_H}\left[1 - \frac{1}{n^2}\right] = \hbar\omega$$

- The change in angular velocity of the electron orbitsphere is identical to the angular velocity of the photon necessary for the excitation, ω_{photon}
- The **correspondence principle holds**

Orbital and Spin Splitting

The ratio of the square of the angular momentum, M^2, to the squre of the energy, U^2, for a pure (l, m) multipole

$$\frac{M^2}{U^2} = \frac{m^2}{\omega^2}$$

The magnetic moment is defined as $\mu = \dfrac{\text{charge x angular momentum}}{2 \text{ x mass}}$

The radiation of a mulipole of order (l, m) carries $m\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$. Thus, the z component of the angular momentum of the corresponding excited state electron orbitsphere is $L_z = m\hbar$

Therefore,

$$\mu_z = \frac{em\hbar}{2m_e} = m\mu_B$$

where μ_B is the Bohr magnton.

The orbital splitting energy is

$$E_{mag}^{orb} = m\mu_B \mathrm{B}$$

Orbital and Spin Splitting cont'd

The spin and orbital splitting energies superimpose; thus, the principal excited state energy levels of the hydrogen atom are split by the energy $E_{mag}^{spin/orb}$

$$E_{mag}^{spin/orb} = m\frac{e\hbar}{2m_e}\mathrm{B} + m_s g\frac{e\hbar}{m_e}\mathrm{B}$$

where

$$n = 2,3,4,...$$
$$\ell = 1,2,...,n-1$$
$$m = -\ell, -\ell+1, ..., 0, ..., +\ell$$
$$m_s = \pm\frac{1}{2}$$

Selection Rules for the Electric Dipole Transition

$$\Delta m = 0, \pm 1$$
$$\Delta m_s = 0$$

Resonant Line Shape

$$\frac{1}{\tau} = \frac{power}{energy}$$

$$\frac{1}{\tau} = \frac{\left[\frac{2\pi c}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)k^{2l+1}|Q_{lm} + Q'_{lm}|^2\right]}{[\hbar\omega]} = 2\pi\left(\frac{e^2}{h}\right)\sqrt{\frac{\varepsilon_o}{\mu_o}}\frac{2\pi}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)\left(\frac{3}{l+3}\right)^2(kna_o)^{2l}\omega$$

$$\mathrm{E}(\omega) = \int_0^\infty e^{-\alpha t}e^{-i\omega t}dt = \frac{1}{\alpha - i\omega}$$

The relationship between the rise-time and the band-width for exponential decay is

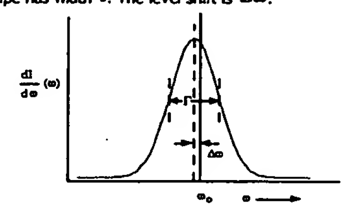
$$\tau\Gamma = \frac{1}{\pi}$$

The energy radiated per unit frequency interval is

$$\frac{dI(\omega)}{d\omega} = I_o\frac{\Gamma}{2\pi}\frac{1}{(\omega - \omega_o - \Delta\omega)^2 + (\Gamma/2)^2}$$

Broadening of the Spectral Line and the Radiative Reaction

Broadening of the spectral line due to the rise-time and shifting of the spectral line due to the radiative reaction. The resonant line shape has width Γ. The level shift is $\Delta\omega$.



Lamb Shift

The Lamb Shift of the $^2P_{1/2}$ state of the hydrogen atom is due to conservation of linear momentum of the electron, atom, and photon.

Electron Component

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{E_{2\pi}}{h} = 3\frac{(E_{h\nu})^2}{h2m_ec^2} = 1052\ MHz$$

where $E_{h\nu}$ is

$$E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{1}{|X_{lm}|^2_{l=1}} - h\Delta f$$

$$E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{3}{8\pi} - h\Delta f;$$

$$h\Delta f <<< 1$$

$$\therefore \quad E_{h\nu} = 13.6\left(1 - \frac{1}{n^2}\right)\frac{3}{8\pi}$$

Lamb Shift cont'd

Atom Component

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{E_{h\nu}}{h} = \frac{1}{2}\frac{(E_{h\nu})^2}{2m_Hc^2} = 6.5\ MHz$$

Sum of the Components

$$\Delta f = 1052\ MHz + 6.5\ MHz = 1058.5\ MHz$$

The experimental Lamb Shift is 1058 MHz.

Instability of Excited States

$n=2,3,4,\ldots,$

$$\sigma_{photon}=\frac{e}{4\pi(r_n)^2}\left[Y_0^0(\theta,\phi)-\frac{1}{n}\left[Y_0^0(\theta,\phi)+Y_\ell^m(\theta,\phi)\mathrm{Re}\left[1+e^{i\omega_n t}\right]\right]\right]\delta(r-r_n)$$

$$\sigma_{electron}=\frac{-e}{4\pi(r_n)^2}\left[Y_0^0(\theta,\phi)+Y_\ell^m(\theta,\phi)\mathrm{Re}\left[1+e^{i\omega_n t}\right]\right]\delta(r-r_n)$$

$$\sigma_{photon}+\sigma_{electron}=\frac{e}{4\pi(r_n)^2}$$

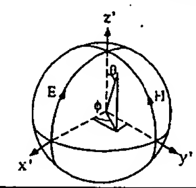
$$\left[Y_0^0(\theta,\phi)\dot{\delta}(r-r_n)-\frac{1}{n}Y_0^0(\theta,\phi)\delta(r-r_n)-\left(1+\frac{1}{n}\right)\left[Y_\ell^m(\theta,\phi)\mathrm{Re}\left[1+e^{i\omega_n t}\right]\right]\delta(r-r_n)\right]$$

Excited states are radiative since spacetime harmonics of $\frac{\omega_n}{c}=k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}}=k$ do exist for which the spacetime Fourier transform of the current density function is nonzero.

Photon Equations

The time-averaged angular-momentum density, $\mathbf{m}$, of the emitted photon is $\mathbf{m}=\frac{1}{8\pi}\mathrm{Re}[\mathbf{r}\times(\mathbf{E}\times\mathbf{B}^*)]=\hbar$

The Cartesian coordinate system wherein the first great circle magnetic field line lies in the yz-plane, and the second great circle electric field line lies in the xz-plane is designated the photon orbitsphere reference frame of a photon orbitsphere.



Nested Set of Great Circle Field Lines Generates the Photon Function

H Field:

$$\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix}=\begin{bmatrix}\cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha)\\0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha)\\\sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha)\end{bmatrix}\begin{bmatrix}x_1'\\y_1'\\z_1'\end{bmatrix}$$

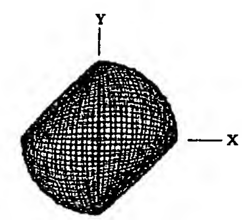
and $\Delta\alpha'=-\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}}\Delta\alpha=\sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}}|\Delta\alpha'|=\sqrt{2}\pi$

E Field:

$$\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix}=\begin{bmatrix}\cos(\Delta\alpha) & -\sin^2(\Delta\alpha) & -\sin(\Delta\alpha)\cos(\Delta\alpha)\\0 & \cos(\Delta\alpha) & -\sin(\Delta\alpha)\\\sin(\Delta\alpha) & \cos(\Delta\alpha)\sin(\Delta\alpha) & \cos^2(\Delta\alpha)\end{bmatrix}\begin{bmatrix}x_2'\\y_2'\\z_2'\end{bmatrix}$$

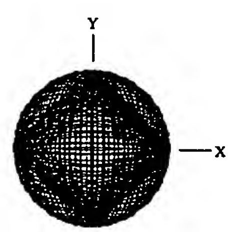
and $\Delta\alpha'=-\Delta\alpha$ replaces $\Delta\alpha$ for $\sum_{n=1}^{\frac{\sqrt{2}\pi}{\Delta\alpha}}\Delta\alpha=\sqrt{2}\pi$; $\sum_{n=1}^{\frac{\sqrt{2}\pi}{|\Delta\alpha'|}}|\Delta\alpha'|=\sqrt{2}\pi$

The Field Line Pattern from the Perspective of Looking along the Z-Axis of a Right-Handed Circularly-Polarized Photon



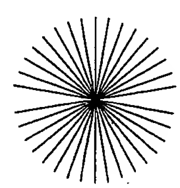
VIEW ALONG THE Z AXIS

The Field Line Pattern from the Perspective of Looking along the Z-Axis of a Linearly-Polarized Photon

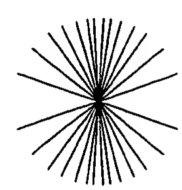


VIEW ALONG THE Z AXIS

Electric Field of a Moving Point Charge $v={}^1/_3c$



Electric Field of a Moving Point Charge $v={}^4/_5c$



The Photon Equation in the Lab Frame of a Right-Handed Circularly-Polarized Photon Orbitsphere

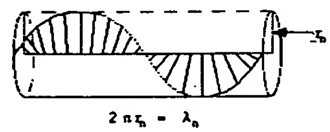
$$\mathbf{E} = \mathbf{E}_0[\mathbf{x} + i\mathbf{y}]e^{-\beta_z z}e^{-j\omega t}$$

$$\mathbf{H} = \left(\frac{\mathbf{E}_0}{\eta}\right)[\mathbf{y} - i\mathbf{x}]e^{-\beta_z z}e^{-j\omega t} = \mathbf{E}_0\sqrt{\frac{\varepsilon}{\mu}}[\mathbf{y} - i\mathbf{x}]e^{-\beta_z z}e^{-j\omega t}$$

with a wavelength of $\lambda = 2\pi\frac{c}{\omega}$

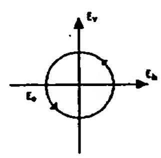
The relationship between the photon orbitspere radius and wavelength is $2\pi r_n = \lambda_n$

The Electric Field Lines of a Right-Handed Circularly-Polarized Photon Orbitsphere



The electric field lines of a right-handed circularly polarized photon orbitsphere as seen along the axis of propagation in the lab inertial reference frame as it passes a fixed point.

The Electric Field Rotation



The electric field rotation of a right-handed circularly polarized photon orbitsphere as seen transverse to the axis of propagation in the lab inertial reference frame as it passes a fixed point.

Elliptically Polarized Photons

Magnitude of the magnetic and electric field lines vary as a function of angular position (ϕ,θ) on the orbitsphere.

$$\mathbf{E}_{\phi,\theta} = \frac{e}{4\pi\varepsilon_o r_n^2}\left(-1 + \frac{1}{n}\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)\left[1 + e^{i\omega_n t}\right]\right\}\right]\right)\delta\left(r - \frac{\lambda}{2\pi}\right);$$

$\omega_n = 0$ for $m = 0$

r_n is the radius of the photon orbitsphere which is equal to $\Delta n a_H$, the change in electron orbitsphere radius.

λ is the photon wavelength which is equal to $\Delta\lambda\frac{c}{\Delta v}$, where $\Delta\lambda$ is the change in orbitsphere de Broglie wavelength and Δv is the change in velocity of the orbitsphere.

$\omega = \frac{2\pi c}{\lambda}$ is the photon angular velocity which is equal to $\Delta\omega$, the change in orbitsphere angular velocity.

Spherical Wave

Photons superimpose, and the amplitude due to N photons is

$$\mathbf{E}_{total} = \sum_{n=1}^{N}\frac{e^{-ik_n|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}f(\phi,\theta)$$

In the far field, the emitted wave is a spherical wave

$$\mathbf{E}_{total} = E_o\frac{e^{-ikr}}{r}$$

- The Green Function is given as the solution of the wave equation. Thus, the superposition of photons gives the classical result.
- As r goes to infinity, the spherical wave becomes a plane wave.
- The double slit interference pattern is predicted.
- From the equation of a photon, the wave-particle duality arises naturally.
- The energy is always given by Planck's equation; yet, an interference pattern is observed when photons add over time or space.

Equations of the Free Electron

Mass Density Function of a Free Electron is a two dimensional disk having the mass density distribution in the $xy(\rho)$-plane

$$\rho_m(\rho,\phi,z) = \frac{m_e}{\frac{2}{3}\pi\rho_0^3}\pi\left(\frac{\rho}{2\rho_o}\right)\sqrt{\rho_0^2 - \rho^2}\,\delta(z)$$

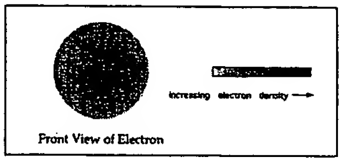
Charge Density Distribution, $\rho_e(\rho,\phi,z)$, in the xy-plane

$$\rho_e(\rho,\phi,z) = \frac{e}{\frac{2}{3}\pi\rho_0^3}\pi\left(\frac{\rho}{2\rho_o}\right)\sqrt{\rho_0^2 - \rho^2}\,\delta(z)$$

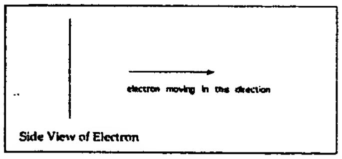
The wave-particle duality arises naturally.

Consistent with scattering experiments.

Front and Side View of Free Electron



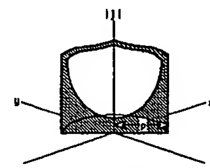
$$\rho_0 = \frac{\hbar}{m v_z}$$



The front view of the magnitude of the mass (charge)density function in the xy-plane of a free electron; side-view of a free electron along the axis of propagation—z-axis.

Current-Density Function

$$\mathbf{J}(\rho,\phi,z,t) = \left[\pi\left(\frac{\rho}{2\rho_o}\right)\frac{e}{\frac{4}{3}\pi\rho_0{}^3}\frac{\hbar}{m_e\sqrt{\rho_0{}^2-\rho^2}}\,\mathbf{i}_\phi\right] + \frac{e\hbar}{m_e\rho_0}\delta\left(z-\frac{\hbar}{m_e\rho_0}t\right)\mathbf{i}_z$$



The magnitude of the current-density function, $|\mathbf{J}|$, of the free electron in the xy-plane cutaway through the top and side.

Angular Momentum

$$\mathbf{Li}_z = \int_0^{2\pi}\int_0^{\rho_0}\pi\left(\frac{\rho}{2\rho_o}\right)\frac{m_e}{\frac{4}{3}\pi\rho_0{}^3}\frac{\hbar}{m_e\sqrt{\rho_0{}^2-\rho^2}}\rho^2\rho\, d\rho\, d\phi$$

$$\mathbf{Li}_z = \hbar$$

Nonradiation Condition

$$\frac{e}{\frac{4}{3}\pi\rho_0{}^3}\frac{\hbar}{m_e}\mathrm{sinc}(2\pi s\bar{\rho}_e) + 2\pi e\frac{\hbar}{m_e\rho_0}\delta(\omega - \mathbf{k}_z \bullet \mathbf{v}_z)$$

$$\mathbf{J}_\perp \propto \mathrm{sinc}(s\bar{\rho}_e) = \frac{\sin 2\pi s\bar{\rho}_e}{2\pi s\bar{\rho}_e}$$

$$2\pi\bar{\rho}_e = \lambda_e$$

Consider the radial wave vector of the sinc function, when the radial projection of the velocity is c

$$\mathbf{s}\bullet\mathbf{v} = \mathbf{s}\bullet\mathbf{c} = \omega_0$$

The relativistically corrected wavelength is

$$\bar{\rho}_e = \lambda_e$$

$$\lambda_e = \frac{h}{m_e v_z} = 2\pi\rho_e$$

Stern-Gerlach Experiment

$$\mathbf{L}_{xy} = \mathbf{L}_z = \hbar\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\hbar}{2}$$

$$\langle \mathbf{L}_z \rangle = \frac{\hbar}{2}$$

$$\langle \mathbf{L}_{xy} \rangle^2_{\Sigma m} + \mathbf{S}^2 = \hbar^2$$

$$\mathbf{S} = \sqrt{\left(1-\frac{1}{4}\right)\hbar^2} = \pm\sqrt{\frac{3}{4}}\hbar$$

S rotates about the z-axis; thus, $\langle \mathbf{S}_z \rangle$, the time averaged projection of the angular momentum onto the axis of the applied magnetic field is $\pm\frac{\hbar}{2}$.

Two Electron Atoms

Central Force Balance Equation with Nonradiation Condition

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r_2^3}\sqrt{s(s+1)}$$

$$r_2 = r_1 = a_0\left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)}\right);\ s = \frac{1}{2}$$

Two Electron Atoms cont'd

Ionization Energies Calculated Using the Poynting Power Theorem

For helium, which has no electric field beyond r_1

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)$$

where $$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1}$$

$$E(magnetic) = \frac{2\pi\mu_o e^2\hbar^2}{m_e^2 r_1^3}$$

Where

For $3 \leq Z$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy$$

The Calculated Energies for Some Two-Electron Atoms

Atom	r_1 (a_o)	Electric Energy (eV)	Magnetic Energy (eV)	Calculated Ionization Energy(eV)	Experimental[a] Ionization Energy (eV)
He	0.567	-23.96	0.63	24.59	24.59
Li^+	0.356	-76.41	2.54	75.56	75.64
Be^{2+}	0.261	-156.08	6.42	154.48	153.89
B^{3+}	0.207	-262.04	12.09	260.35	259.37
C^{4+}	0.171	-398.08	22.83	393.19	392.08
N^{5+}	0.146	-558.20	38.74	552.05	552.06
O^{6+}	0.127	-748.59	55.35	739.67	739.32
F^{7+}	0.113	-962.17	79.37	953.35	953.89

The Calculated Energies for Some Two-Electron Atoms cont'd

Atom	r_1 (a_o)	Electric Energy (eV)	Magnetic Energy (eV)	Calculated Ionization Energy(eV)	Experimental[a] Ionization Energy (eV)
Ne^{8+}	0.101	-1204.9	109.5	1194	1195.80
Na^{9+}	0.0921	-1474.6	148.4	1452	1465.14
Mg^{10+}	0.0843	-1771.0	180.7	1756	1761.80
Al^{11+}	0.0778	-2096.2	243.2	2077	2086.00
Si^{12+}	0.0722	-2447.0	304.8	2420	2437.70
P^{13+}	0.0673	-2826.3	278.3	2801	2817.04
S^{14+}	0.0631	-3232.1	488.4	3204	3223.95
Cl^{15+}	0.0593	-3665.0	546.3	3633	3658.58
Ar^{16+}	0.0560	-4126.2	651.9	4089	4120.92
K^{17+}	0.0530	-4612.5	767.7	4572	4611.11

[a] from theoretical calculations for ions Ne^{8+} to K^{17+}

Elastic Electron Scattering from Helium Atoms

Aperture distribution function, $a(\rho,\phi,z)$, for the scattering of an incident electron plane wave $\pi(z)$

by the He atom

$$\frac{2}{4\pi(0.567a_o)^2}[\delta(r-0.567a_o)]$$

is

$$a(\rho,\phi,z) = \pi(z) \otimes \frac{2}{4\pi(0.567a_o)^2}[\delta(r-0.567a_o)]$$

$$a(\rho,\phi,z) = \frac{2}{4\pi(0.567a_o)^2}\sqrt{(0.567a_o)^2 - z^2}\,\delta(r - \sqrt{(0.567a_o)^2 - z^2})$$

Far Field Scattering (Circular Symmetry)

$$F(s) = \frac{2}{4\pi(0.567a_o)^2} 2\pi \int_0^\infty \int_{-\infty}^\infty \sqrt{(0.567a_o)^2 - z^2}\,\delta(\rho - \sqrt{(0.567a_o)^2 - z^2})J_o(s\rho)e^{-iwz}\rho\, d\rho\, dz$$

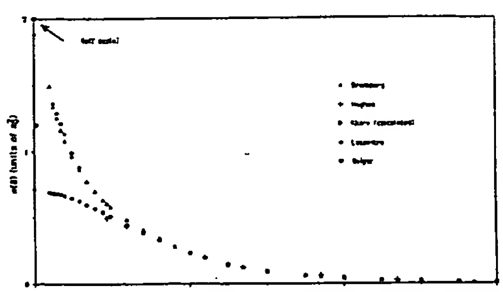
$$I_1^{sc} = F(s)^2$$

$$= I_e \left\{ \left[\frac{2\pi}{(z_o w)^2 + (z_o s)^2}\right]^{\frac{1}{2}} \left\{ 2\left[\frac{z_o s}{(z_o w)^2 + (z_o s)^2}\right] j_{1/2}\left[((z_o w)^2 + (z_o s)^2)^{1/2}\right] - \left[\frac{z_o s}{(z_o w)^2 + (z_o s)^2}\right]^2 j_{3/2}\left[((z_o w)^2 + (z_o s)^2)^{1/2}\right] \right\} \right\}^2$$

$$s = \frac{4\pi}{\lambda}\sin\frac{\theta}{2};\ w = 0\ (\text{units of } \AA^{-1})$$

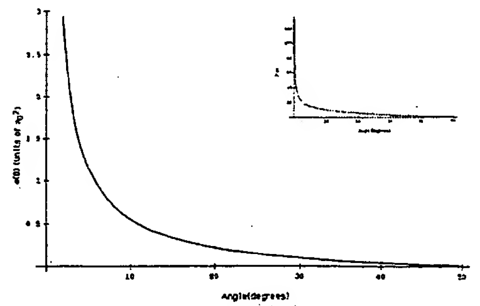
Experimental Results

and Born Approximation



The experimental results of Bromberg, the extrapolated experimental data of Hughes, the small angle data of Geiger, and the semiexperimental results of Lassettre for the elastic differential cross section for the elastic scattering of electrons by helium atoms and the elastic differential cross section as a function of angle numerically calculated by Khare using the first Born approximation and first-order exchange approximation.

The Closed Form Function



The closed form function for the elastic differential cross section for the elastic scattering of electrons by helium atoms. The scattering amplitude function, F(s), is shown as an insert.

The Nature of the Chemical Bond of Hydrogen

The Laplacian in ellipsoidal coordinates is solved with the constraint of nonradiation

$$(\eta-\zeta)R_\xi\frac{\delta}{\delta\xi}(R_\xi\frac{\delta\phi}{\delta\xi})+(\zeta-\xi)R_\eta\frac{\delta}{\delta\eta}(R_\eta\frac{\delta\phi}{\delta\eta})+(\xi-\eta)R_\zeta\frac{\delta}{\delta\zeta}(R_\zeta\frac{\delta\phi}{\delta\zeta})=0$$

The Force Balance Equation for the Hydrogen Molecule

$$\frac{\hbar^2}{m_e a^2 b^2}2ab^2X=\frac{e^2}{4\pi\varepsilon_o}X+\frac{\hbar^2}{2m_e a^2 b^2}2ab^2X \quad \text{where} \quad X=\frac{1}{\sqrt{\xi+a^2}}\frac{1}{\xi+b^2}\frac{1}{c}\sqrt{\frac{\xi^2-1}{\xi^2-\eta}}$$

has the parametric solution $r(t)=\mathbf{i}a\cos\omega t+\mathbf{j}b\sin\omega t$

When the **Semimajor Axis, a, is** $a=a_o$

The Nature of the Chemical Bond of Hydrogen cont'd

The Internuclear Distance, $2c'$, which is the distance between the foci is

$$2c'=\sqrt{2}a_o$$

The experimental internuclear distance is $\sqrt{2}a_o$.

The Semiminor Axis is $b=\frac{1}{\sqrt{2}}a_o$

The Eccentricity, e, is $e=\frac{1}{\sqrt{2}}$

The Energies of the Hydrogen Molecule

The Potential Energy of the Two Electrons in the Central Field of the Protons at the Foci

$$V_e=\frac{-2e^2}{8\pi\varepsilon_o\sqrt{a^2-b^2}}\ln\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}=-67.813\ eV$$

The Potential Energy of the Two Protons

$$V_p=\frac{e^2}{8\pi\varepsilon_o\sqrt{a^2-b^2}}=19.23\ eV$$

The Kinetic Energy of the Electrons

$$T=\frac{\hbar^2}{2m_e a\sqrt{a^2-b^2}}\ln\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}=33.906\ eV$$

The Energy, V_m, of the Magnetic Force Between the Electrons

$$V_m=\frac{-\hbar^2}{4m_e a\sqrt{a^2-b^2}}\ln\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}=-16.9533\ eV$$

The Energies of the Hydrogen Molecule cont'd

The Total Energy

$$E_T=V_e+T+V_m+V_p$$

$$E_T=-13.6\ eV\left[\left(2\sqrt{2}-\sqrt{2}+\frac{\sqrt{2}}{2}\right)\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}-\sqrt{2}\right]=-31.63\ eV$$

The Energy of Two Hydrogen Atoms

$$E(2H[a_H])=-27.21\ eV$$

The Bond Dissociation Energy, E_D, is the difference between the total energy of the corresponding hydrogen atoms and E_T.

$$E_D=E(2H[a_H])-E_T=4.43\ eV$$

The experimental energy

E_D=4.45 eV

Proton and Neutron

The proton and neutron each comprise three charged fundamental particles called quarks and three massive photons called gluons.

Proton Parameters

$$\lambda_{C,p}=\lambda_{c,q}=\frac{2\pi a_0 m_e}{\alpha^{-1}m_p}=1.3\ X\ 10^{-15}\ m=r_p=r_q$$

$$m_p=m_q+m_g^*=m_q^*$$

$$m_q=\frac{m_p}{2\pi}$$

$$m_q^*=2\pi m_q=2\pi\ X\ \frac{m_p}{2\pi}=m_p$$

$$m_g^*=m_p-m_q=m_p\left[1-\frac{1}{2\pi}\right]$$

$$E=m_qc^2+m_gc^2=\frac{m_p}{2\pi}c^2+m_p\left[1-\frac{1}{2\pi}\right]c^2=m_pc^2$$

m_p proton rest mass

$\lambda_{C,p}$ is the Compton wavelength of the proton

$\lambda_{c,q}$ is the Compton wavelength bar of the quarks

r_p is the radius of the proton

r_q is the radius of the quarks

m_q is the rest mass of the quarks

m_g^* is the relativistic mass of the gluons

m_q^* is the relativistic mass of the quarks

Proton and Neutron cont'd

Neutron Parameters

$$\lambda_{C,n} = \lambda_{c,q} = \frac{2\pi m_e m_q}{\alpha^{-1} m_N} = 1.3214 \ X \ 10^{-15} \ m = r_n = r_q$$

$$m_N = m_q + m_g^* = m_q^*$$

$$m_q = \frac{m_N}{2\pi}$$

$$m_q^* = 2\pi m_q = 2\pi \ X \ \frac{m_N}{2\pi} = m_N$$

$$m_g^* = m_N - m_q = m_N\left[1 - \frac{1}{2\pi}\right]$$

$$E = m_q c^2 + m_g c^2 = \frac{m_N}{2\pi}c^2 + m_N\left[1 - \frac{1}{2\pi}\right]c^2 = m_N c^2$$

m_N neutron rest mass

$\lambda_{C,n}$ is the Compton wavelength of the neutron

$\lambda_{c,q}$ is the Compton wavelength bar of the quarks

r_n is the radius of the neutron

r_q is the radius of the quarks

m_q is the rest mass of the quarks

m_g^* is the relativistic mass of the gluons

m_q^* is the relativistic mass of the quarks

Quark and Gluon Functions of the Proton

The proton functions can be viewed as a linear combination of three fundamental particles, three quarks, of charge $+\frac{2}{3}e$, $+\frac{2}{3}e$, and $-\frac{1}{3}e$. Each quark is associated with its gluon where the quark mass/charge function has the same angular dependence as the gluon mass/charge function.

The quark mass function of a proton is

$$\frac{m_P}{2\pi}\left[\frac{1}{3}(1+\sin\theta\sin\phi) + \frac{1}{3}(1+\sin\theta\cos\phi) + \frac{1}{3}(1+\cos\theta)\right]$$

The charge function of the quarks of a proton is

$$e\left[\frac{2}{3}(1+\sin\theta\sin\phi) + \frac{2}{3}(1+\sin\theta\cos\phi) - \frac{1}{3}(1+\cos\theta)\right]$$

The radial electric field of a proton is

$$E_r = \frac{-\alpha^{-1}e}{4\pi\varepsilon_o r^3}\frac{2\pi m_e}{\frac{m_P}{m_e}\alpha^{-1}}\left[\frac{3}{2}(1+\sin\theta\sin\phi) + \frac{3}{2}(1+\sin\theta\cos\phi) - 3(1+\cos\theta)\right]$$

Quark and Gluon Functions of the Neutron

The neutron functions can be viewed as a linear combination of three fundamental particles, three quarks, of charge $+\frac{2}{3}e$, $-\frac{1}{3}e$, and $-\frac{1}{3}e$. Each quark is associated with its gluon where the quark mass/charge function has the same angular dependence as the gluon mass/charge function.

The quark mass function of a neutron is

$$\frac{m_N}{2\pi}\left[\frac{1}{3}(1+\sin\theta\sin\phi) + \frac{1}{3}(1+\sin\theta\cos\phi) + \frac{1}{3}(1+\cos\theta)\right]$$

The charge function of the quarks of a neutron is

$$e\left[\frac{2}{3}(1+\sin\theta\sin\phi) - \frac{1}{3}(1+\sin\theta\cos\phi) - \frac{1}{3}(1+\cos\theta)\right]$$

The radial electric field of a neutron is

$$E_r = \frac{-\alpha^{-1}e}{4\pi\varepsilon_o r^3}\frac{2\pi m_e}{\frac{m_N}{m_e}\alpha^{-1}}\left[\frac{3}{2}(1+\sin\theta\sin\phi) - 3(1+\sin\theta\cos\phi) - 3(1+\cos\theta)\right]$$

Magnetic Moments

Proton Magnetic Moment

$$\mu = \frac{\text{charge X angular momentum}}{2 \text{ X mass}}$$

$$\mu_{proton} = \frac{\frac{2}{3}e\frac{2}{3}\hbar}{2\frac{m_p}{2\pi}} = \frac{4}{9}2\pi\frac{e\hbar}{2m_p} = 2.79253\mu_N$$

where μ_N is the nuclear magneton $\frac{e\hbar}{2m_p}$

The experimental magnetic moment of the proton is 2.79268 μ_N

Neutron Magnetic Moment

The magnetic moment of the neutron, μ_n, is

$$\mu_n = \left[1 - \frac{4}{9}2\pi - \frac{3}{25}\right]\mu_N = -1.91253\mu_N$$

The experimental magnetic moment of the neutron is −1.91315 μ_n

Beta Decay Energy

The nuclear reaction for the beta decay of a neutron is

$$^1n \rightarrow {}^1H + \beta + \bar{v}_e + 0.7835 \ MeV$$

where $\bar{v}_e$ is the electron antineutrino. The energy terms of the beta decay are

$$E_{mag} = m_e c^2 \frac{\alpha}{2\pi} = 1.09 \ X \ 10^6 \ eV \qquad E_{ele} = \frac{e^2}{8\pi\varepsilon_o r_1} = 5.46 \ X \ 10^3 \ eV$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m_e\hbar^2}{\left[\frac{m_p}{2\pi}\right]^2 r_1^2} = 2.553 \ X \ 10^3 \ eV \qquad E_{mag}(gluon) = \left[\frac{3}{25}\right]^2 E_{mag} = 15.7 \ X \ 10^3 \ eV$$

The beta decay energy is

$$E_\beta = E_{mag} - E_{mag}(gluon) - E_{ele} + T$$

$$E_\beta = 1.09 \ X \ 10^6 - 15.7 \ X \ 10^3 - 5.46 \ X \ 10^3 + 2.553 \ X \ 10^3$$

$$E_\beta = 0.7836 \ MeV$$

Maxwell's Equations and Special Relativity

Maxwell's equations and special relativity are based on the law of propagation of a electromagnetic wave front in the form

$$\frac{1}{c^2}\left(\frac{\delta\omega}{\delta t}\right)^2 - \left[\left(\frac{\delta\omega}{\delta x}\right)^2 + \left(\frac{\delta\omega}{\delta y}\right)^2 + \left(\frac{\delta\omega}{\delta z}\right)^2\right] = 0$$

For any kind of wave advancing with limiting velocity and capable of transmitting signals, the equation of front propagation is the same as the equation for the front of a light wave.

Thus, the equation

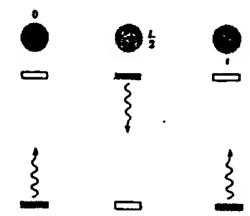
$$\frac{1}{c^2}\left(\frac{\delta\omega}{\delta t}\right)^2 - (grad\,\omega)^2 = 0$$

acquires a general character; it is more general than Maxwell's equations from which Maxwell originally derived it.

The Classical Wave Equation Governs:

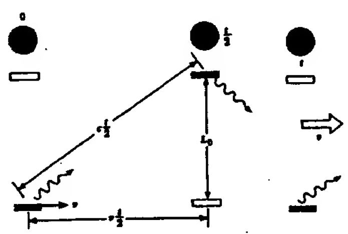
- The motion of bound electrons
- The propagation of any form of energy
- Measurements between inertial frames of reference such as time, mass, momentum, and length (**Minkowski tensor**)
- A relativistic correction of spacetime due to particle production or annihilation (**Schwarzschild metric**)
- Fundamental particle production and the conversion of matter to energy
- The expansion and contraction of the Universe
- The basis of the relationship between Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and special, and general relativity

A Light-Pulse Clock at Rest on the Ground As Seen by an Observer on the Ground



The dial represents a conventional clock on the ground.

A Light-Pulse Clock in a Spacecraft As Seen by an Observer on the Ground



The mirrors are parallel to the direction of the motion of the spacecraft.
The dial represents a conventional clock on the ground.

Time Interval Relation Between Ticks t of the Moving Clock and L_0, the Vertical Distance Between the Mirrors

$$\left(c\frac{t}{2}\right)^2 = L_0^2 + \left(v\frac{t}{2}\right)^2 \qquad t = \frac{\frac{2L_0}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

But $\frac{2L_0}{c}$ is the time t_0 interval between ticks on the clock on the ground, and so the

time dilation relationship based on the constant maximum speed of light in any inertial frame is $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

wherein the parameters are:
t_0 = time interval on clock at rest relative to an observer
t = time interval on clock in motion relative to an observer
v = speed of relative motion
c = speed of light

Minkowski Tensor $\eta_{\mu\nu}$

The Metric $g_{\mu\nu}$ for Euclidean Space Called the Minkowski Tensor $\eta_{\mu\nu}$ is

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{c^2} & 0 & 0 \\ 0 & 0 & \frac{1}{c^2} & 0 \\ 0 & 0 & 0 & \frac{1}{c^2} \end{pmatrix}$$

In this case, the separation of proper time between two events x^μ and is $x^\mu + dx^\mu$

$$d\tau^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$$

The Equivalence of the Gravitational Mass and the Inertial Mass

Experimentally $\frac{m_g}{m_i}$ = universal constant
which is predicted by Newton's Law of mechanics and gravitation.

The energy equation of Newtonian gravitation

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \text{constant}$$

Since h, the angular momentum per unit mass, is

$$h = L/m = |\mathbf{r} \times \mathbf{v}| = r_0 v_0 \sin\phi$$

Eccentricity e may be written as

$$e = [1 + \left(v_0^2 - \frac{2GM}{r_0}\right)\frac{r_0^2 v_0^2 \sin^2\phi}{G^2 M^2}]^{1/2}$$

m is the inertial mass of a particle
v_0 is the speed of the particle
r_0 is the distance of the particle from a massive object
ϕ is the angle between the direction of motion of the particle and the radius vector from the object
M is the total (including a particle) of mass of the object

Classification of the Orbits

The eccentricity e given by Newton's differential equations of motion in the case of the central field permits the classification of the orbits.

According to the total energy E:

$E<0, \; e<1$	ellipse
$E<0, \; e=0$	circle (special case of ellipse)
$E=0, \; e=1$	parabolic orbit
$E>0, \; e>1$	hyperbolic obit

Classification of the Orbits cont'd

According to the orbital velocity relative to the gravitational velocity squared $\frac{2GM}{r_0}$:

$v_0^2 < \frac{2GM}{r_0}$	$e<1$	ellipse
$v_0^2 < \frac{2GM}{r_0}$	$e=0$	circle (special case of ellipse)
$v_0^2 = \frac{2GM}{r_0}$	$e=1$	parabolic orbit
$v_0^2 > \frac{2GM}{r_0}$	$e>1$	hyperbolic orbit

Continuity Conditions for the Production of a Particle From a Photon Traveling at Light Speed

- A photon traveling at the speed of light gives rise to a particle with an initial **radius equal to its Compton wavelength bar**

$$r = \lambda_C = \frac{\hbar}{mc}$$

- The particle must have an **orbital velocity equal to Newtonian gravitational escape velocity** v_g **of the antiparticle**

$$v_g = \sqrt{\frac{2Gm}{r}} = \sqrt{\frac{2Gm_0}{\lambda_C}}$$

- The eccentricity is one
- The orbital energy is zero
- The particle production trajectory is a parabola relative to the center of mass of the antiparticle

A Gravitational Field As a Front Equivalent to a Light Wave Front

The particle with a finite gravitational mass gives rise to a gravitational field that travels out as a front equivalent to a light wave front

The form of the outgoing gravitational field front traveling at the speed of light is

$$f\left(t - \frac{r}{c}\right)$$

and $d\tau^2$ is given by

$$d\tau^2 = f(r)dt^2 - \frac{1}{c^2}\left[f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

The Speed of Light

The speed of light as a constant maximum as well as phase matching and continuity conditions of the electromagnetic and gravitational waves require the following form of the squared displacements:

$$(c\tau)^2 + (v_g t)^2 = (ct)^2$$

$$\tau^2 = t^2\left(1 - \left(\frac{v_g}{c}\right)^2\right)$$

Thus,

$$f(r) = \left(1 - \left(\frac{v_g}{c}\right)^2\right)$$

In order that the wave front velocity does not exceed c in any frame, spacetime must undergo time dilation and length contraction due to the particle production event.

*The derivation and result of spacetime time dilation is analogous to the derivation and result of **special relativistic time dilation** wherein the relative velocity of two inertial frames replaces the gravitational velocity.*

Quadratic Form Of The Infinitesimal Squared Temporal Displacement

General form of the metric due to the relativistic effect on spacetime due to mass m_0

$$d\tau^2 = \left(1 - \left(\frac{v_g}{c}\right)^2\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \left(\frac{v_g}{c}\right)^2\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

Gravitational radius, r_g, of each orbitsphere of the particle production event, each of mass m

$$r_g = \frac{2Gm}{c^2}$$

$$d\tau^2 = \left(1 - \frac{r_g}{r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

Masses and their effects on spacetime **superimpose**. The separation of proper time between two events and x^μ is $x^\mu + dx^\mu$

$$d\tau^2 = \left(1 - \frac{2GM}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

The Origin of Gravity

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the curvature of spacetime and is the origin of gravity.

The metric $g_{\mu\nu}$ for non-euclidean space due to the relativistic effect on spacetime due to mass m_0 is

$$g_{\mu\nu} = \begin{pmatrix} -\left(1-\frac{2Gm_0}{c^2 r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{c^2}\left(1-\frac{2Gm_0}{c^2 r}\right)^{-1} & 0 & 0 \\ 0 & 0 & \frac{1}{c^2}r^2 & 0 \\ 0 & 0 & 0 & \frac{1}{c^2}r^2 \sin^2\theta \end{pmatrix}$$

Particle Production Continuity Conditions

- The photon to particle event requires a transition state that is continuous.
- The velocity of a transition state orbitsphere is the speed of light.
- The radius, r_n, is the Compton wavelength bar, λ_C

$$\lambda_C = \frac{\hbar}{m_0 c} = r_\alpha^*$$

- The Planck equation energy, the electric potential energy, and the magnetic energy are equal to m_0c^2.

The Masses of Fundamental Particles

The Schwarzschild metric gives the relationship whereby matter causes relativistic corrections to spacetime that determines the masses of fundamental particles.

Substitution of $r = \lambda_C;\ dr = d\theta = 0;\ \sin^2\theta = 1$ into the Schwarzchild metric gives

$$d\tau = dt\left(1 - \frac{2Gm_0}{c^2 r_\alpha^*} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

with, $v^2 = c^2$

$$\tau = ti\sqrt{\frac{2GM}{c^2 r_\alpha^*}} = ti\sqrt{\frac{2GM}{c^2 \lambda_c}} = ti\frac{v_g}{c}$$

Relationship of the Equivalent Particle Production Energies

When the orbitsphere velocity is the speed of light:

Continuity conditions based on the constant maximum speed of light given by **Maxwell's equations:**

(Mass energy = Planck equation energy = electric potential energy = magnetic energy = mass/spacetime metric energy)

$$m_0c^2 = \hbar\omega^* = V = E_{mag} = E_{spacetime}$$

$$m_0c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0\lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_0)^2\lambda_C^3} = \frac{\alpha h}{1\ \mathrm{sec}}\sqrt{\frac{\lambda_C c^2}{2Gm}}$$

Continuity Conditions Based on the Constant Maximum Speed of Light Given by the Schwarzschild Metric

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2\lambda_C}}}{\alpha} = i\frac{v_g}{\alpha c}$$

Masses of Fundamental Particles

- **Each of the Planck equation energy, electric energy, and magnetic energy corresponds to a particle given by the relationship between the proper time and the coordinate time.**
- The **electron and antielectron** correspond to the **Planck equation energy.**
- The **muon and antimuon** correspond to the **electric energy.**
- The **tau and antitau** correspond to the **magnetic energy.**
- The particle must possess the escape velocity v_g relative to the antiparticle where $v_g < c$.
- According to Newton's law of gravitation, the eccentricity is one and the particle production trajectory is a parabola relative to the center of mass of the antiparticle.

The Electron-Antielectron Lepton Pair

A clock is defined in terms of a self-consistent system of units used to measure the particle mass.

$$2\pi\frac{\hbar}{mc^2}=\sec\sqrt{\frac{2Gm^2}{c\alpha^2\hbar}}$$

$$m_e=\left(\frac{h\alpha}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{c\hbar}{2G}\right)^{\frac{1}{4}}=9.1097\ X\ 10^{-31}\ kg$$

$$m_e=9.1097\ X\ 10^{-31}\ kg-18\ eV(\nu_e)=9.1094\ X\ 10^{-31}\ kg$$

$$m_{e\ experimental}=9.1095\ X\ 10^{-31}\ kg$$

The Muon-Antimuon Lepton Pair

$$2\pi\frac{\hbar}{mc^2}=2\pi\sec\sqrt{\frac{2Gm_1\alpha^2m}{c\hbar}}$$

The mass of the muon/antimuon is

$$m_\mu=\frac{\hbar}{c}\left(\frac{1}{2Gm_e(\alpha\sec)^3}\right)^{\frac{1}{3}}=1.8902\ X\ 10^{-28}\ kg$$

The muon/antimuon mass is corrected for the experimental mass/energy deficit of the $0.25\ MeV$ neutrino.

$$m_\mu=1.890563\ X\ 10^{-28}\ kg-0.25\ MeV(\nu_\mu)=1.8857\ X\ 10^{-28}\ kg$$

$$m_{\mu experimental}=1.8836\ X\ 10^{-28}\ kg$$

The Tau-Antitau Lepton Pair

$$2\pi\frac{\hbar}{mc^2}=2\sec\sqrt{\frac{2Gm_1(2\pi)^3\alpha^4m}{c\hbar}}$$

The mass of the tau/antitau is

$$m_\tau=\frac{\hbar}{c}\left(\frac{1}{2Gm_e}\right)^{\frac{1}{2}}\left(\frac{1}{2\sec\alpha^2}\right)^{\frac{2}{3}}=3.17\ X\ 10^{-27}\ kg$$

The tau/antitau mass is corrected for the experimental mass/energy deficit of the $17\ keV$ neutrino.

$$m_\tau=3.17\ X\ 10^{-27}\ kg-17\ keV(\nu_\tau)=3.17\ X\ 10^{-27}\ kg$$

$$m_{\tau\ experimental}=3.17\ X\ 10^{-27}\ kg$$

Down-Down-Up Neutron (DDU)

$$2\pi\frac{2\pi\hbar}{\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]c^2}=\sec\sqrt{\frac{2G\left[\frac{m_N}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]\right]^2}{3c(2\pi)^2\hbar}}$$

The neutron mass is

$$m_{N\ calculated}=(3)(2\pi)\left(\frac{1}{1-\alpha}\right)\left(\frac{2\pi\hbar}{\sec c^2}\right)^{\frac{1}{2}}\left(\frac{2\pi(3)c\hbar}{2G}\right)^{\frac{1}{4}}$$

$$=1.6744X10^{-27}\ kg$$

$$m_{N experimental}=1.6749X10^{-27}\ kg$$

Strange-Strange-Charmed Neutron (SSC)

$$2\pi\frac{2\pi\hbar}{\frac{m_{ssc}}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]c^2}=2\pi\sec\sqrt{\frac{2G\alpha^2m_{\Lambda\Lambda}\left[\frac{m_{ssc}}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]\right]}{3c(2\pi)^2\hbar}}$$

The strange-strange-charmed neutron mass is

$$m_{ssc\ calculated}=(3)(2\pi)\left(\frac{1}{1-\alpha}\right)\left(\frac{\hbar}{\sec c^2}\right)^{\frac{1}{3}}\left(\frac{2\pi(3)c\hbar}{2m_{\Lambda\Lambda}G\alpha^2}\right)^{\frac{1}{3}}$$

$$m_{ssc\ calculated}=4.90X10^{-27}\ kg=2.75\ GeV/c^2$$

The observed mass of the Ω^- hyperon that contains three strange quarks (sss) is

$$m_{\Omega^-}=1673\ MeV/c^2$$

Strange-Strange-Charmed Neutron (SSC) cont'd

Thus, an estimate for the dynamical mass of the strange quark, m_s, is

$$m_s=\frac{m_{\Omega^-}}{3}=\frac{1673\ MeV/c^2}{3}=558\ MeV/c^2$$

The dynamical mass of the charmed quark, m_c, has been determined by fitting quarkonia spectra; and from the observed masses of the charmed pseudoscalar mesons $D^0(1865)$ and $D^+(1869)$.

$$m_c=1.580\ GeV/c^2$$

Thus,

$$m_{ssc\ experimental}=2m_s+m_c=2(558\ MeV/c^2)+1673\ MeV/c^2$$

$$m_{ssc\ experimental}=2.79\ GeV/c^2$$

Bottom-Bottom-Top Neutron (BBT)

$$2\pi \frac{2\pi\hbar}{\frac{m_{bbt}}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]c^2} = 2\sec\sqrt{\frac{2G\alpha^4 m_{u/d}\left[\frac{m_{bbt}}{3}\left[\frac{1}{2\pi}-\frac{\alpha}{2\pi}\right]\right]}{3c2(2\pi)^2\hbar}}$$

The bottom-bottom-top neutron mass is

$$m_{bbt\ calculated} = (3)(2\pi)\left(\frac{1}{1-\alpha}\right)\left(\frac{2\pi\hbar}{2\sec c^2}\right)^{\frac{2}{3}}\left(\frac{2\pi(3)c\hbar}{m_{u/d}G\alpha^4}\right)^{\frac{1}{3}}$$

$$m_{bbt\ calculated} = 3.48\ X\ 10^{-25}\ kg = 195\ GeV\ /\ c^2$$

The dynamical mass of the bottom quark, M_B, has been determined by fitting quarkonia spectra; and from the observed masses of the bottom pseudoscalar mesons $B^0(5275)$ and $B^+(5271)$.

$$m_b = 4.580\ GeV/c^2$$

Bottom-Bottom-Top Neutron (BBT) cont'd

Thus, the predicted dynamical mass of the top quark based on the dynamical mass of the bottom quark is

$$m_{t\ calculated} = m_{bbt\ calculated} - 2m_b = 195\ GeV\ /\ c^2 - 2(4.580\ GeV\ /\ c^2)$$

$$m_{t\ calculated} = 186\ GeV\ /\ c^2$$

From about 21 top quark events, the CDF collaboration calculates the mass of the top quark as

$$176 \pm 13\ GeV\ /\ c^2$$

From about 17 top quark events, the D_0 collaboration calculates the mass of the top quark as

$$199 \pm 30\ GeV\ /\ c^2$$

Gravitational Potential Energy

The gravitational radius, α_G or r_G , of an orbitsphere of mass m_0 is defined as

$$\alpha_G = r_G = \frac{Gm_0}{c^2}$$

When the $r_G = r_\alpha^* = \lambda_C$, the gravitational potential energy equals m_0c^2

$$\frac{Gm_0}{c^2} = \lambda_c = \frac{\hbar}{m_0 c}$$

$$\frac{Gm_0^2}{\lambda_C} = \frac{Gm_0^2}{r_\alpha^*} = \hbar\omega^* = m_0c^2$$

Gravitational Potential Energy cont'd

The mass m_0 is the Planck mass, m_U ,

$$m_Uc^2 = \hbar\omega^* = V = E_{mag} = \frac{Gm_U^2}{\lambda_C^*}$$

$$m_U = m_0 = \sqrt{\frac{\hbar c}{G}}$$

The corresponding gravitational velocity, v_G , is defined as

$$v_G = \sqrt{\frac{Gm_0}{\lambda_C}} = \sqrt{\frac{Gm_0}{\lambda_C}}$$

Relationship of the Equivalent Planck Mass Particle Production Energies

(Mass energy = Planck equation energy = electric potential energy = magnetic Energy = gravitational potential energy = mass/spacetime metric energy)

$$m_0c^2 = \hbar\omega^* = V = E_{mag} = E_{grav} = E_{spacetime}$$

$$m_0c^2 = \hbar\omega^* = \frac{\hbar^2}{m_0\lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_0)^2\lambda_C^3} = \alpha^{-1}\frac{\mu_0 e^2 c^2}{2h}\sqrt{\frac{Gm_0}{\lambda_C}}\sqrt{\frac{\hbar c}{G}} = \frac{\alpha h}{1\sec}\sqrt{\frac{\lambda_C c^2}{2Gm}}$$

Equivalent energies give the particle masses in terms of the gravitational velocity, v_G, and the Planck mass, m_U

$$m_0 = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\frac{\sqrt{\frac{Gm_0}{\lambda_C}}}{c}m_0 = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\sqrt{\frac{Gm_0}{c^2\lambda_C}}m_0 = \alpha^{-1}\frac{\mu_0 e^2 c}{2h}\frac{v_G}{c}m_0 = \frac{v_G}{c}m_0$$

Planck Mass Particles

- A pair of particles each of the **Planck mass** corresponding to the **gravitational potential energy** is not observed since the velocity of each transition state orbitsphere is the **gravitational velocity** v_G that in this case **is the speed of light; whereas, the Newtonian gravitational escape velocity v_g is twice the speed of light.**
- In this case, an electromagnetic wave of mass energy equivalent to the Planck mass travels in a **circular orbit** about the center of mass of another electromagnetic wave of mass energy equivalent to the Planck mass wherein the **eccentricity is equal to zero** and the **escape velocity can never be reached.**

Planck Mass Particles cont'd

- The Planck mass is a **"measuring stick."** The extraordinarily high Planck mass ($\sqrt{\frac{\hbar c}{G}} = 2.18\ X\,10^{-8}\ kg$) is the unobtainable mass bound imposed by the angular momentum and speed of the photon relative to the gravitational constant.
- It is analogous to the unattainable bound of the speed of light for a particle possessing finite rest mass imposed by the Minkowski tensor.

Astrophysical Implications of Planck Mass Particles

- The limiting speed of light **eliminates the singularity** problem of Einstein's equation that arises as the radius of a black hole equals the Schwarzschild radius.
- When the **gravitational potential energy density** of a massive body such as a **blackhole** equals that of a particle having the Planck mass, the **matter may transition to photons of the Planck mass.**
- Even light from a black hole will escape when the decay rate of the trapped matter with the concomitant spacetime expansion is greater than the effects of gravity which oppose this expansion.

Astrophysical Implications of Planck Mass Particles cont'd

- Gamma-ray bursts are the most energetic phenomenon known that can release an explosion of gamma-rays packing 100 times more energy than a Supernova explosion.
- The annihilation of a black hole may be the source of γ-ray bursts.
- The source may be due to **conversion of matter to photons of the Planck mass/energy, which may also give rise to cosmic rays.**
- According to the GZK cutoff, the cosmic spectrum cannot extend beyond $5\ X\,10^{19}\ eV$, but AGASA, the world's largest air shower array, has shown that the spectrum is extending beyond $10^{20}\ eV$ without any clear sign of cutoff. **Photons each of the Planck mass may be the source of these inexplicably energetic cosmic rays.**

The Schwarzschild Metric Gives the Relationship Whereby Matter Causes Relativistic Corrections to Spacetime

- The limiting velocity c results in the **contraction of spacetime due to particle production.** The contraction is given by $2\pi r_g$ where r_g is the gravitational radius of the particle. This has implications for the expansion of spacetime when matter converts to energy
- Q **The mass/energy to expansion/contraction quotient** of spacetime is given by the ratio of the mass of a particle at production divided by T the period of production.

$$Q = \frac{m_0}{T} = \frac{m_0}{\frac{2\pi r_g}{c}} = \frac{m_0}{\frac{2\pi \frac{2Gm_0}{c^2}}{c}} = \frac{c^3}{4\pi G} = 3.22\ X\,10^{34}\ \frac{kg}{sec}$$

- The gravitational equations with the equivalence of the particle production energies permit the **conservation of mass/energy** ($E = mc^2$) **and spacetime** ($\frac{c^3}{4\pi G} = 3.22\ X\,10^{34}\ \frac{kg}{sec}$).

Cosmological Consequences

- The ***Universe is closed*** (It is ***finite but with no boundary***).
- The **Universe is a 3-sphere Universe**-Riemannian three dimensional hyperspace plus time of constant positive curvature at each r-sphere.
- ***The Universe is oscillatory in matter/energy and spacetime*** with a finite minimum radius, the gravitational radius.
- Spacetime expands as mass is released as energy which provides the basis of the **atomic, thermodynamic, and cosmological arrows of time.**
- Different regions of space are **isothermal** even though they are separated by greater distances than that over which light could travel during the time of the expansion of the Universe.

Cosmological Consequences cont'd

- Presently, **stars exist which are older than the elapsed time of the present expansion** as stellar evolution also occurred during the contraction phase.
- The **maximum power** radiated by the Universe which occurs at the beginning of the expansion phase is

$$P_U = \frac{c^5}{4\pi G} = 2.89\ X\,10^{51}\ W$$

- **Observations beyond the beginning** of the expansion phase are **not possible** since the Universe is entirely matter filled.

The Period of Oscillation Based on Closed Propagation of Light

- Conservation of mass/energy during harmonic expansion and contraction
- The gravitational potential energy E_{grav}

$$E_{grav} = \frac{Gm_U^2}{r}$$

  is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G.

- The gravitational velocity v_G is the speed of light in a circular orbit wherein the eccentricity is equal to zero and the escape velocity from the Universe can never be reached.
- The period of the oscillation of the Universe and the period for light to transverse the Universe corresponding to the gravitational radius r_G must be equal.

The Period of Oscillation Based on Closed Propagation of Light cont'd

- The harmonic oscillation period, T, is

$$T = \frac{2\pi r_G}{c} = \frac{2\pi G m_U}{c^3} = \frac{2\pi G(2 X 10^{54}\ kg)}{c^3} = 3.10 X 10^{19}\ sec = 9.83 X 10^{11}\ years$$

  where the mass of the Universe, m_U, is approximately $2 X 10^{54}\ kg$ (The initial mass of the Universe of $2 X 10^{54}\ kg$ is based on internal consistency with the size, age, Hubble constant, temperature, density of matter, and power spectrum of the Universe.)
- Thus, the observed **Universe will expand as mass is released as photons for** $4.92 X 10^{11} years$**. At this point in its world line, the Universe will obtain its maximum size and begin to contract.**

The Differential Equation of the Radius of the Universe

- Simple harmonic oscillator having a restoring force, F, which is proportional to the radius.
- The proportionality constant, k, is given in terms of the potential energy, E, gained as the radius decreases from the maximum expansion to the minimum contraction.

$$\frac{E}{\aleph^2} = k$$

- The gravitational potential energy E_{grav}

$$E_{grav} = \frac{Gm_U^2}{r}$$

- Is equal to $m_U c^2$ when the radius of the Universe r is the gravitational radius r_G.

$$F = -k\aleph = -\frac{m_U c^2}{r_G^2}\aleph = -\frac{m_U c^2}{\left(\frac{Gm_U}{c^2}\right)^2}\aleph$$

The Differential Equation of the Radius of the Universe $\aleph$, Is

$$m_U \ddot{\aleph} + \frac{m_U c^2}{r_G^2}\aleph = 0$$

$$m_U \ddot{\aleph} + \frac{m_U c^2}{\left(\frac{Gm_U}{c^2}\right)^2}\aleph = 0$$

The *Maximum Radius of the Universe*

The *Maximum Radius of the Universe*, the amplitude, r_0, of the time harmonic variation in the radius of the Universe, is given by the quotient of the total mass of the Universe and, the mass/energy to expansion/contraction quotient.

$$r_0 = \frac{m_U}{Q} = \frac{m_U}{\frac{c^3}{4\pi G}}$$

$$r_0 = \frac{2 X 10^{54}\ kg}{\frac{c^3}{4\pi G}} = 1.97 X 10^{12}\ light\ years$$

The *Minimum Radius*

The *Minimum Radius* corresponds to the gravitational radius

$$r_g = \frac{2Gm_U}{c^2}$$

$$r_g = \frac{2G(2 X 10^{54}\ kg)}{c^2} = 3.12 X 10^{11}\ light\ years$$

When the gravitational radius r_g *is the radius of the Universe, the proper time is equal to the coordinate time by*

$$\tau = ti\sqrt{\frac{2GM}{c^2 r_a^*}} = ti\sqrt{\frac{2GM}{c^2 \lambda_c}} = ti\frac{v_g}{c}$$

And the gravitational escape velocity v_g *of the Universe is the speed of light.*

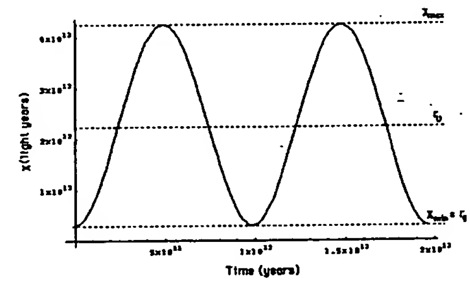
The Radius of the Universe As a Function of Time

$$\aleph = \left(r_s + \frac{cm_U}{Q}\right) - \frac{cm_U}{Q}\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)$$

$$\aleph = \left(\frac{2Gm_U}{c^2} + \frac{cm_U}{\frac{c^3}{4\pi G}}\right) - \frac{cm_U}{\frac{c^3}{4\pi G}}\cos\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)$$

$$\aleph = 2.28\ X\ 10^{12} - 1.97\ X\ 10^{12}\cos\left(\frac{2\pi t}{9.83\ X\ 10^{11}\ yrs}\right)\ light\ years$$

The Radius of the Universe as a Function of Time

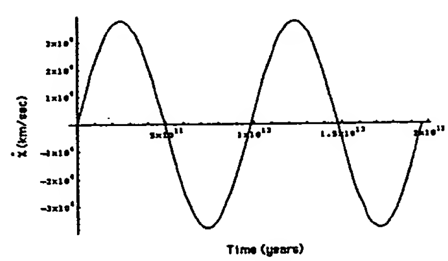


The Expansion/Contraction Rate, $\dot{\aleph}$

$$\dot{\aleph} = 4\pi c X 10^{-3}\sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec}$$

$$\dot{\aleph} = 3.77\ X\ 10^{6}\sin\left(\frac{2\pi t}{9.83\ X\ 10^{11}\ yrs}\right)\frac{km}{sec}$$

The Expansion/Contraction Rate of the Universe As a Function of Time



The Hubble Constant

The ***Hubble Constant*** is given by the ratio of the expansion rate given in units of $\frac{km}{sec}$ divided by the radius of the expansion in *Mpc*. The radius of expansion is equivalent to the radius of the light sphere with an origin at the time point when the Universe stopped contracting and started to expand. The radius is the time of expansion $t\ Mpc$.

$$H = \frac{\dot{\aleph}}{t\ Mpc} = \frac{4\pi c\ X\ 10^{-3}\sin\left(\frac{2\pi t}{\frac{2\pi G m_U}{c^3}}\right)\frac{km}{sec}}{t\ Mpc}$$

$$H = \frac{\dot{\aleph}}{t} = \frac{3.77\ X\ 10^{6}\sin\left(\frac{2\pi t}{3.01\ X\ 10^{5}\ Mpc}\right)\frac{km}{sec}}{t\ Mpc}$$

The Hubble Constant cont'd

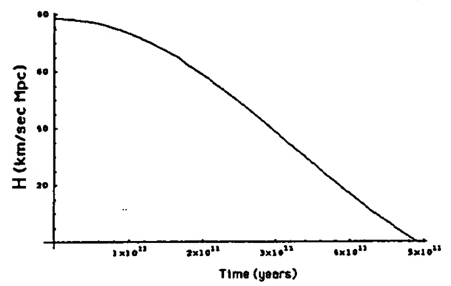
For , $t = 10^{10}\ light\ years = 3.069\ X\ 10^{3}\ Mpc$ the Hubble, H_0 , constant is

$$H_0 = 78.6\ \frac{km}{sec \cdot Mpc}$$

The experimental value is

$$H_0 = 80 \pm 17\ \frac{km}{sec \cdot Mpc}$$

The Hubble Constant of the Universe As a Function of Time



The Density of the Universe As a Function of Time

- The density of the Universe as a function of time $\rho_U(t)$ is given by the ratio of the mass as a function of time and the volume as a function of time.

$$\rho_U(t)=\frac{m_U(t)}{V(t)}=\frac{m_U(t)}{\frac{4}{3}\pi\Re(t)^3}=\frac{\frac{m_U}{2}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)}{\frac{4}{3}\pi\left(\left(\frac{2Gm_U}{c^2}+\frac{\frac{cm_U}{c^3}}{4\pi G}\right)-\frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)^3}$$

$$\rho_U(t)=\frac{1\ X\ 10^{57}\left(1+\cos\left(\frac{2\pi t}{9.83\ X\ 10^{11}\ yrs}\right)\right)g}{\frac{4}{3}\pi\left(2.16\ X\ 10^{30}-1.86\ X\ 10^{30}\cos\left(\frac{2\pi t}{9.83\ X\ 10^{11}\ yrs}\right)cm\right)^3}$$

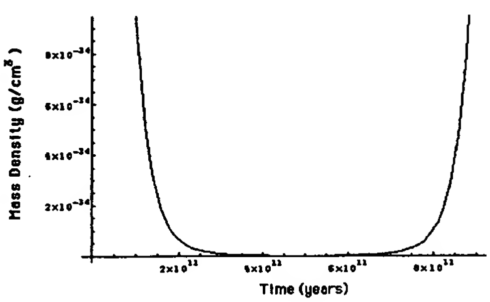
- For $t=10^{10}$ *light years* $=3.069\ X\ 10^3\ Mpc$

$$\rho_U=1.7\ X\ 10^{-32}\ g/cm^3$$

- The density of luminous matter of stars and gas of galaxies is about

$$\rho_U=2\ X\ 10^{-31}\ g/cm^3$$

The Density of the Universe As a Function of Time



The Power of the Universe As a Function of Time, $P_U(t)$

$$P_U(t)=\frac{c^5}{8\pi G}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)\right)$$

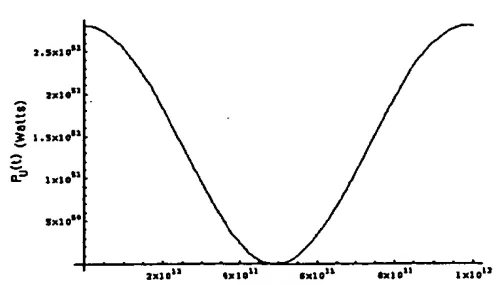
$$P_U(t)=1.45\ X\ 10^{51}\left(1+\cos\left(\frac{2\pi t}{9.83\ X\ 10^{11}\ yrs}\right)\right)W$$

For $t=10^{10}$ *light years*

$$P_U(t)=2.88\ X\ 10^{51}\ W$$

The observed power is consistent with that predicted.

The Power of the Universe As a Function of Time



The Temperature of the Universe as a Function of Time Follows from the Stefan-Boltzman Law

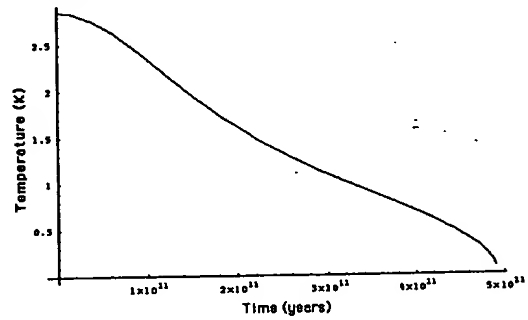
$$T_U(t)=\left(\frac{1}{1+\frac{Gm_U(t)}{c^2\Re(t)}}\right)\left[\frac{R_U(t)}{e\sigma}\right]^{\frac{1}{4}}=\left(\frac{1}{1+\frac{Gm_U(t)}{c^2\Re(t)}}\right)\left[\frac{\frac{P_U(t)}{4\pi\Re(t)^2}}{e\sigma}\right]^{\frac{1}{4}}$$

$$T_U(t)=\left[\frac{1}{1+\frac{\frac{Gm_U}{2}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)}{c^2\left(\left(\frac{2Gm_U}{c^2}+\frac{\frac{cm_U}{c^3}}{4\pi G}\right)-\frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)}}\right]x\left[\frac{\frac{\frac{c^5}{8\pi G}\left(1+\cos\left(\frac{2\pi t}{\frac{2\pi r_G}{c}}\right)\right)}{4\pi\left(\left(\frac{2Gm_U}{c^2}+\frac{\frac{cm_U}{c^3}}{4\pi G}\right)-\frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi t}{\frac{2\pi Gm_U}{c^3}}\right)\right)^2}}{e\sigma}\right]^{\frac{1}{4}}$$

The Temperature of the Universe As a Function of Time – cont'd

$$T_U(t) = \left[\left(\frac{1}{1 + \dfrac{0.74 \, X \, 10^{27} \left(1 + \cos\left(\dfrac{2\pi t}{9.83 \, X \, 10^{11} \, yrs}\right)\right) m}{\left[2.16 \, X \, 10^{23} - 1.86 \, X \, 10^{23} \cos\left(\dfrac{2\pi t}{9.83 \, X \, 10^{11} \, yrs}\right) m\right]}} \right) X \left[\frac{\dfrac{1.45 \, X \, 10^{34} \left(1 + \cos\left(\dfrac{2\pi t}{9.83 \, X \, 10^{11} \, yrs}\right)\right) W}{4\pi \left[2.16 \, X \, 10^{23} - 1.86 \, X \, 10^{23} \cos\left(\dfrac{2\pi t}{9.83 \, X \, 10^{11} \, yrs}\right) m\right]^2}}{5.67 X 10^{-8} \, W m^{-2} K^{-4}} \right]^{\frac{1}{4}} \right]$$

The Temperature of the Universe As a Function of Time During the Expansion Phase



Power Spectrum of the Cosmos

- The power spectrum of the cosmos, as measured by the **LAS CAMPANAS SURVEY**, generally follows the prediction of cold dark matter on the scales of 200 million to 600 million light-years.
- However, the power increases dramatically on scales of **600 million to 900 million light-years.**
- The infinitesimal temporal displacement, $d\tau^2$, is

$$d\tau^2 = \left(1 - \frac{2Gm_U}{c^2 r}\right) dt^2 - \frac{1}{c^2}\left[\left(\frac{dr^2}{1 - \dfrac{2Gm_U}{c^2 r}}\right) + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

- The relationship between the proper time and the coordinate time is

$$\tau = t\sqrt{1 - \frac{2Gm_U}{c^2 r}} \qquad \tau = t\sqrt{1 - \frac{r_s}{r}}$$

Power Spectrum of the Cosmos cont'd

- The power maximum in the proper frame occurs at

$$\tau = 5 \, X \, 10^9 \, light \, years \sqrt{1 - \frac{3.12 \, X \, 10^{11} \, light \, years}{3.22 \, X \, 10^{11} \, light \, years}}$$

$$\tau = 880 \, X \, 10^6 \, light \, years$$

- The power maximum of the current observable Universe is predicted to occur on the scale of $880 \, X 10^6 \, light \, years$.
- There is excellent agreement between the predicted value and the experimental value $600\text{-}900 \, X 10^6 \, light \, years$.

The Expansion/ Contraction Acceleration, $\ddot{\aleph}$

$$\ddot{\aleph} = 2\pi \frac{c^4}{Gm_U} \cos\left(\frac{2\pi t}{\dfrac{2\pi Gm_U}{c^3} \, sec}\right)$$

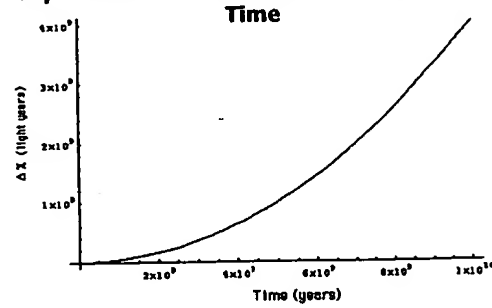
$$\ddot{\aleph} = H_o = 78.7 \cos\left(\frac{2\pi t}{3.01 \, X \, 10^5 \, Mpc}\right) \frac{km}{sec \cdot Mpc}$$

- The differential in the radius of the Universe $\Delta\aleph$ due to its acceleration is given by

$$\Delta\aleph = 1/2 \ddot{\aleph} t^2$$

- The differential in expanded radius for the elapsed time of expansion, $= 10^{10} \, light \, years = 3.069 \, X \, 10^3 \, Mpc$ corresponds to a decease in brightness of a supernovae standard candle of about an order of magnitude of that expected where the distance is taken as $\Delta\aleph$. This result based on **the predicted rate of acceleration of the expansion is consistent with the experimental observation**
- **The microwave background radiation image** obtained by the **BOOMERANG telescope** was consistent with a Universe of nearly flat geometry since the commencement of its expansion. **The data is consistent with a large offset radius of the Universe with a fractional increase in size since the commencement of expansion about 10 billion years ago.**

The Differential Expansion of the Light Sphere Due to the Acceleration of the Expansion of the Cosmos As a Function of Time



The Periods of Spacetime Expansion/Contraction and Particle Decay/Production for the Universe Are Equal

- The period of the expansion/contraction cycle of the radius of the Universe, T , is

$$T = \frac{2\pi G m_U}{c^3} \text{ sec}$$

- It follows from the Poynting power theorem with spherical radiation that the transition lifetimes are given by the ratio of energy and the power of the transition.

$$\tau = \frac{energy}{power}$$

$$\tau = \frac{[\hbar\omega]}{\left[\frac{2\pi c}{[(2l+1)!!]^2}\left(\frac{l+1}{l}\right)k^{2l+1}|Q_{lm}+Q'_{lm}|^2\right]} = \frac{1}{2\pi}\left(\frac{h}{e^2}\right)\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{[(2l+1)!!]^2}{2\pi}\left(\frac{l}{l+1}\right)\left(\frac{l+3}{3}\right)^2\frac{1}{(kr_n)^{2l}\omega}$$

- Exponential decay applies to electromagnetic energy decay.

$$h(t) = e^{-\frac{1}{T}t}u(t)$$

The Coordinate Time Is Imaginary Because Energy Transitions Are Spacelike Due Spacetime Expansion From Matter to Energy Conversion

- For example, the mass of the electron (a fundamental particle) is given by

$$\frac{2\pi\lambda_C}{\sqrt{\frac{2Gm_e}{\lambda_C}}} = \frac{2\pi\lambda_C}{v_g} = i\alpha^{-1}\text{sec}$$

where v_g is Newtonian gravitational velocity.

- When the gravitational radius r_g is the radius of the Universe, the proper time is equal to the coordinate time by

$$\tau = it\sqrt{\frac{2GM}{c^2 r_\alpha^*}} = it\sqrt{\frac{2GM}{c^2\lambda_c}} = it\frac{v_g}{c}$$

and the gravitational escape velocity v_g of the Universe is the speed of light.

- Replacement of the coordinate time, t , by the spacelike time, it , gives

$$h(t) = e^{-i\frac{1}{T}t} = \cos\frac{2\pi}{T}t$$

where the period is T.

Period Equivalents

The periods of spacetime expansion/contraction and particle decay/production for the Universe are equal because only the particles which satisfy Maxwell's equations and the relationship between proper time and coordinate time imposed by the Schwarzschild metric may exist.

Continuity conditions based on the constant maximum speed of light **(Maxwell's equations)**

$$m_e c^2 = \hbar\omega^* = V = E_{mag} = E_{spacetime}$$

$$m_e c^2 = \hbar\omega^* = \frac{\hbar^2}{m_e\lambda_C^2} = \alpha^{-1}\frac{e^2}{4\pi\varepsilon_0\lambda_C} = \alpha^{-1}\frac{\pi\mu_0 e^2\hbar^2}{(2\pi m_e)^2\lambda_C^3} = \frac{\alpha h}{1\text{ sec}}\sqrt{\frac{\lambda_C c^2}{2Gm}}$$

Continuity conditions based on the constant maximum speed of light **(Schwarzschild metric)**

$$\frac{\text{proper time}}{\text{coordinate time}} = \frac{\text{gravitational wave condition}}{\text{electromagnetic wave condition}} = \frac{\text{gravitational mass phase matching}}{\text{charge/inertial mass phase matching}}$$

$$\frac{\text{proper time}}{\text{coordinate time}} = i\frac{\sqrt{\frac{2Gm}{c^2\lambda_C}}}{\alpha} = i\frac{v_g}{\alpha c}$$

Wave Equation

$$\frac{1}{c^2}\left(\frac{\delta\omega}{\delta t}\right)^2 - (grad\omega)^2 = 0$$

The equation of the radius of the Universe, $\aleph$, may be written as

$$\aleph = \left(\frac{2Gm_U}{c^2} + \frac{\frac{cm_U}{c^3}}{4\pi G}\right) - \frac{\frac{cm_U}{c^3}}{4\pi G}\cos\left(\frac{2\pi}{\frac{2\pi Gm_U}{c^3}\text{ sec}}\left(t - \frac{\aleph}{c}\right)\right) m$$

which is a solution to the wave equation.

Conclusion

Maxwell's equations, Planck's equation, the de Broglie equation, Newton's laws, and Special, and General Relativity are Unified.

The Grand Unified Theory of Classical Quantum Mechanics

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